

# MATHEMATICS NOTES

## FORM 2

### CUBES AND CUBE ROOTS

#### CHAPTER TWENTY FOUR

#### Specific Objectives

By the end of the topic the learner should be able to:

- a) Find the cube of a number by multiplication
- b) Find the cube root of a number by factor method
- c) Find cubes of numbers from mathematical tables
- d) Evaluate expressions involving cubes and cube roots
- e) Apply the knowledge of cubes and cube roots to real life situations

#### Content

- a.) Cubes of numbers by multiplication.
- b.) Cube roots of numbers by factor method.
- c.) Cubes from mathematical tables.
- d.) Expressions involving cubes and cube roots
- e.) Application of cubes and cube roots

## Introduction Cubes

The cube of a number is simply a number multiplied by itself three times e.g.

$$a \times a \times a = a^3$$

$$1 \times 1 \times 1 = 1^3;$$

$$8 = 2 \times 2 \times 2 = 2^3;$$

$$27 = 3 \times 3 \times 3 = 3^3;$$

### Example 1

What is the value of  $6^3$ ?

$$6^3 = 6 \times 6 \times 6$$

$$= 36 \times 6$$

$$= 216$$

### Example 2

Find the cube of 1.4

$$= 1.4 \times 1.4 \times 1.4$$

$$= 1.96 \times 1.4$$

$$= 2.744$$

## Use of tables to find roots

The cubes can be read directly from the tables just like squares and square root.

Cube Roots using factor methods

Cubes and cubes roots are opposite. The cube root of a number is the number that is multiplied by itself three times to get the given number

### Example

The cube root of 64 is written as;

$$\sqrt[3]{64} = 4 \quad \text{Because } 4 \times 4 \times 4 = 64$$

$$\sqrt[3]{27} = 3 \quad \text{Because } 3 \times 3 \times 3 = 27$$

### Example

Evaluate:  $\sqrt[3]{216}$

$$= \sqrt[3]{(2 \times 2 \times 2 \times 3 \times 3 \times 3)}$$

$$= 2 \times 3$$

$$= 6$$

### Note;

After grouping them into pairs of three you chose one number from the pair and multiply

### Example

Find:

The volume of a cube is  $1000 \text{ cm}^3$ . What is the length of the cube

Volume of the cube,  $v = l^3$

$$L^3 = 1000$$

$$L = \sqrt[3]{1000}$$

$$= 10$$

The length of the cube is therefore 10 cm

End of topic

Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

## CHAPTER TWENTY FIVE

### RECIPROCAL

#### Specific Objectives

By the end of the topic the learner should be able to:

- Find reciprocals of numbers by division
- Find reciprocals of numbers from tables
- Use reciprocals of numbers in computation.

## Content

- a.) Reciprocals of numbers by division
- b.) Reciprocals of numbers from tables
- c.) Computation using reciprocals

## Introduction

The reciprocal of a number is simply the number put in fraction form and turned upside down e.g., the reciprocal of 2.

## Solution:

Change 2 into fraction form which is  $\frac{2}{1}$ ,

Then turn it upside down and get  $\frac{1}{2}$

## Note:

When you multiply a number by its reciprocal you get 1,

$$\frac{2}{1} \times \frac{1}{2} = 1$$

## Finding the reciprocal of decimals

Finding the reciprocal of a decimal can be done in a number of ways.

Change the decimal to a fraction first.

### Example.

0.25 is  $\frac{25}{100}$  and is equivalent to the fraction  $\frac{1}{4}$ . Therefore its reciprocal would be  $\frac{4}{1}$  or 4.

Keep the decimal and form the fraction  $\frac{1}{??}$  Which can then be or converted to a decimal.

### Example

0.75 The reciprocal is  $\frac{1}{0.75}$ . Using a calculator, the decimal form can be found by performing the operation: 1 divided by 0.75. The decimal reciprocal in this case is a repeating decimal, 1.33333....

After finding a reciprocal of a number, perform a quick check by multiplying your original number and the reciprocal to determine that the product.

## Reciprocal of Numbers from Tables.

Reciprocal of numbers can be found using tables.

### Example

Find the reciprocal of 2.456 using the reciprocal tables.

### Solution.

Using reciprocal tables, the reciprocal of 2.456 is  $0.4082 - 0.0010 = 0.4072$

### Example

Find the reciprocal of 45.8.

### Solution

You first write 45.8 in standard form which is  $4.58 \times 10^1$ .

$$\begin{aligned}\text{Then } \frac{1}{45.8} &= \frac{1}{4.58 \times 10^1} \\ &= \frac{1}{10} \times \frac{1}{4.58} \\ &= \frac{1}{10} \times 0.2183 \\ &= 0.02183\end{aligned}$$

### Example

Find the reciprocal of 0.0236

### Solution

Change 0.0236 in standard form which is  $2.36 \times 10^{-2}$

$$\begin{aligned}\frac{1}{0.0236} &= \frac{1}{2.36 \times 10^{-2}} \\ &= \frac{1}{10^{-2}} \times \frac{1}{2.36} \\ &= 10^2 \times 0.4237 \\ &= 42.37\end{aligned}$$

### Example

Use reciprocal tables to solve the following:

$$\frac{1}{0.0125} + \frac{1}{12.5}$$

### Solution

Multiply the numerators by the reciprocal of denominators, then add them

$$1(\text{reciprocal } 0.0125) + 1(\text{reciprocal } 12.5)$$

Using tables find the reciprocals,

$$= 1(80) + 1(0.08)$$

$$= 80.08$$

### Example

$$\frac{4}{0.375} - \frac{3}{37.5}$$

### Solution

$$= 4(\text{rec}0.375) - 3(37.5)$$

$$= (4 \times 2.667) - (3 \times 0.026667)$$

$$= 10.59$$

End of topic

Did you understand everything?

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## CHAPTER TWENTY SIX

# INDICES AND LOGARITHMS

### Specific Objectives

By the end of the topic the learner should be able to:

- a.) Define indices (powers)
- b.) State the laws of indices
- c.) Apply the laws of indices in calculations
- d.) Relate the powers of 10 to common logarithms
- e.) Use the tables of common logarithms and anti-logarithms in computation.

### Content

- a.) Indices (powers) and base
- b.) Laws of indices (including positive integers, negative integers and fractional indices)
- c.) Powers of 10 and common logarithms
- d.) Common logarithms:
  - ✓ characteristics
  - ✓ mantissa
- e.) Logarithm tables
- f.) Application of common logarithms in multiplication, division, powers and roots.

## Introduction

### Index and base form

The power to which a number is raised is called index or indices in plural.

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2$$

5 is called the power or index while 2 two is the base.

$$100 = 10^2$$

2 is called the index and 10 is the base.

### Laws of indices

For the laws to hold the base must be the same.

#### Rule 1

Any number, except zero whose index is 0 is always equal to 1

#### Example

$$5^0 = 1$$
$$10000000000000000^0 = 1$$

#### Rule 2

To multiply an expression with the same base, copy the base and add the indices.

$$a^m \times a^n = a^{m+n}$$

#### Example

$$5^2 \times 5^3 = 5^5$$
$$= 3125$$

#### Rule 3

To divide an expression with the same base, copy the base and subtract the powers.

$$a^m \div a^n = a^{m-n}$$

#### Example

$$9^5 \div 9^2 = 9^3$$

#### Rule 4

To raise an expression to the nth index, copy the base and multiply the indices

$$a^{mn} = a^{m \times n}$$

## Example

$$(5^3)^2 =$$

$$5^{3 \times 2} = 5^6$$

## Rule 5

When dealing with a negative power, you simply change the power to positive by changing it into a fraction with 1 as the numerator.

$$a^{-m} = \frac{1}{a^m}$$

## Example

$$2^{-2} = \frac{1}{2^2}$$

$$= \frac{1}{4}$$

## Example

Evaluate:

$$\text{a.) } 2^3 \times 2^{-3} = 2^{(3+(-3))}$$

$$= 2^0$$

$$= 1$$

$$\text{b.) } \left(\frac{2}{3}\right)^{-2} = \left(\frac{1}{\frac{2}{3}}\right)^2$$

$$= \left(\frac{1}{\frac{2}{3}}\right)^2$$

$$= 1 \div \frac{2}{3}$$

$$= 1 \times \frac{3}{2} = \frac{3}{2}$$

$$\text{or } \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

## Fractional indices

Fractional indices are written in fraction form. In summary if  $a^{\frac{1}{n}} = b$ ,  $a$  is called the  $n^{\text{th}}$  root of  $b$  written as  $\sqrt[n]{b}$ .

## Example

$$27^{\frac{1}{3}} = \sqrt[3]{27} = 3 \quad 16^{\frac{3}{4}} = (\sqrt[4]{16})^3 = 2^3 = 8$$

$$= 3$$

$$4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}}$$

$$= \frac{1}{\sqrt{4}}$$

## LOGARITHM

Logarithm is the power to which a fixed number (the base) must be raised to produce a given number. In summary the expression  $a^m = n$  is written as  $\log_a n = m$ .

$a^m = n$  is the index notation while  $\log_a n = m$  is the logarithm notation.

### Examples

Index notation	Logarithm form
$2^2 = 4$	$\log_2 4 = 2$
$9^{\frac{1}{2}} = 3$	$\log_9 3 = \frac{1}{2}$
$b^n = m$	$\log_b m = n$

Reading logarithms from the tables is the same as reading squares square roots and reciprocals.

We can read logarithms of numbers between 1 and 10 directly from the table. For numbers greater than 10 we proceed as follows:

Express the number in standard form,  $A \times 10^n$ . Then  $n$  will be the whole number part of the logarithms.

Read the logarithm of  $A$  from the tables, which gives the decimal part of the logarithm. Then add it to  $n$  which is the power of 10 to form the positive part of the logarithm.

## Example

Find the logarithm of:

379

## Solution

379

$$= 3.79 \times 10^2$$

$$\text{Log } 3.79 = 0.5786$$

Therefore the logarithm of 379 is  $2 + 0.5786 = 2.5786$

The whole number part of the logarithm is called the characteristic and the decimal part is the mantissa.

## Logarithms of Positive Numbers less than 1

### Example

Log to base 10 of 0.034

We proceed as follows:

Express 0.034 in standard form, i.e.,  $A \times 10^n$ .

Read the logarithm of A and add to n

$$\text{Thus } 0.034 = 3.4 \times 10^{-2}$$

Log 3.4 from the tables is 0.5315

$$\text{Hence } 3.4 \times 10^{-2} = 10^{0.5315} \times 10^{-2}$$

Using laws of indices add  $0.5315 + -2$  which is written as  $2.5315$ .

It reads bar two point five three one five. The negative sign is written directly above two to show that it's only the characteristic is negative.

### Example

Find the logarithm of:

0.00063

### Solution

$$= 6.3 \times 10^{-3} \text{ (Find the logarithm of } 6.3)$$

$$= 10^{0.7993} \times 10^{-3}$$

$$= 10^{-3} + 0.7993$$

$$= 3.7993$$

## ANTILOGARITHMS

Finding antilogarithm is the reverse of finding the logarithms of a number. For example the logarithm of 1000 to base 10 is 3. So the antilogarithm of 3 is 1000. In algebraic notation, if

Log x = y then antilog of y = x.

### Example

Find the antilogarithm of 2.3031

### Solution

Let the number be x

$$X = 10^{2.3031}$$

$$= 10^{-2+0.3031}$$

$= 10^{-2} \times 10^{0.3031}$  (Find the antilog, press shift and log then key in the number)

$$= 10^{-2} \times 2.01$$

$$= \frac{1}{100} \times 2.01$$

$$= \frac{2.01}{100}$$

$$= 0.0201$$

### Example

Use logarithm tables to evaluate:

$$\frac{456 \times 398}{271}$$

Number	Standard form	logarithm
456	$4.56 \times 10^2$	2.6590
398	$3.98 \times 10^2$	2.5999

5.2589

+

271	$2.71 \times 10^2$	2.4330	-
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$6.697 \times 10^2 \leftarrow 2.8259$

$$= 669.7$$

To find the exact number find the antilog of 2.8259 by letting the characteristic part to be the power of ten then finding the antilog of 0.8259

### Example

Operations involving bar

Evaluate  $\frac{415.2 \times 0.0761}{135}$

### Solution

Number	logarithm
415.2	2.6182
0.0761	<u>2.8814 +</u>
135	1.4996 2.1303 -
$2.341 \times 10^{-1}$	1.3693
0.2341	

### Example

$$\begin{aligned} \sqrt{0.945} &= (9.45 \times 10^{-1})^{\frac{1}{2}} \\ &= (10^{1.9754 \times \frac{1}{2}}) \end{aligned}$$

### Note;

In order to divide 1 .9754 by 2 , we write the logarithm in search away that the characteristic is exactly divisible by 2 .If we are looking for the n<sup>th</sup> root , we arrange the characteristic to be exactly divisible by n)

$$\begin{aligned} 1.9754 &= -1 + 0.9754 \\ &= -2 + 1.9754 \end{aligned}$$

$$\begin{aligned} \text{Therefore, } \frac{1}{2}(1.9754) &= \frac{1}{2}(-2 + 1.9754) \\ &= -1 + 0.9877 \\ &= 1.9877 \end{aligned}$$

Find the antilog of 1.9877 by writing the mantissa as power of 10 and then find the antilog of characteristic.

$$= 9.720 \times 10^{-1}$$

$$= 0.9720$$

### Example

$$\sqrt[3]{0.0618}$$

Number                      logarithm

$$\sqrt[3]{0.0618} = 2.7910 \times \frac{1}{3}$$

$$= (3 + 1.7910) \times \frac{1}{3}$$

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$$3.954 \times 10^{-1} = 1.5970 \text{ (find the antilog)}$$

$$0.3954$$

End of topic

Did you understand everything?

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## Past KCSE Questions on the cubes, cubes roots, Reciprocals indices and logarithms.

1. Use logarithms to evaluate

$$\sqrt[3]{\frac{36.15 \times 0.02573}{1,938}}$$

2. Find the value of x which satisfies the equation.

$$16^{x^2} = 8^{4x-3}$$

3. Use logarithms to evaluate  $\frac{(1934)^2 \times \sqrt{0.00324}}{436}$

4. Use logarithms to evaluate

$$55.9 \div (0.2621 \times 0.01177)^{1/5}$$

5. Simplify  $2^x \times 5^{2x} \div 2^{-x}$

6. Use logarithms to evaluate  $(3.256 \times 0.0536)^{1/3}$

7. Solve for x in the equation  $32^{(x-3)} \div 8^{(x-4)} = 64 \div 2^x$

8. Solve for x in the equations  $\frac{81^{2x} \times 27^x}{9^x} = 729$

9. Use reciprocal and square tables to evaluate to 4 significant figures, the expression:

$$\left( \frac{1}{24.56} \right) + 4.346^2$$

10. Use logarithm tables, to evaluate

$$\frac{\left( \frac{0.032 \times 14.26}{0.006} \right)^{2/3}}{0.006}$$

11. Find the value of x in the following equation  $49^{(x+1)} + 7^{(2x)} = 350$

12. Use logarithms to evaluate  $\frac{(0.07284)^2}{3\sqrt{0.06195}}$

13. Find the value of m in the following equation  $(1/27^m \times (81)^{-1}) = 243$

14. Given that  $P = 3^y$  express the equation  $3^{(2y-1)} + 2 \times 3^{(y-1)} = 1$  in terms of P hence or otherwise find the value of y in the equation  $3^{(2y-1)} + 2 \times 3^{(y-1)} = 1$

15. Use logarithms to evaluate  $55.9 \div (0.2621 \times 0.01177)^{1/5}$

16. Use logarithms to evaluate

$$\frac{\left( \frac{6.79 \times 0.3911}{\text{Log } 5} \right)^{3/4}}{\text{Log } 5}$$

17. Use logarithms to evaluate

$$\sqrt[3]{1.23 \times 0.0089}$$

79.54

18. Solve for x in the equation

$$X = \sqrt[0.0056]{\frac{1}{2}}$$

$$1.38 \times 27.42$$

## CHAPTER TWENTY SEVEN

### GRADIENT AND EQUATIONS OF STRAIGHT LINES

#### Specific Objectives

By the end of the topic the learner should be able to:

- Define gradient of a straight line
- Determine the gradient of a straight line through known points
- Determine the equation of a straight line using gradient and one known point
- Express a straight line equation in the form  $y = mx + c$

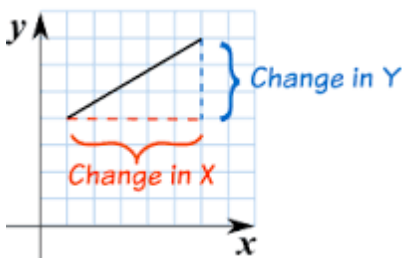
- e.) Interpret the equation  $y = mx + c$
- f.) Find the x- and y- intercepts from an equation of a line
- g.) Draw the graph of a straight line using gradient and x- and y- intercepts
- h.) State the relationship of gradients of perpendicular lines
- i.) State the relationship of gradients of parallel lines
- j.) Apply the relationship of gradients of perpendicular and parallel lines to get equations of straight lines.

### Content

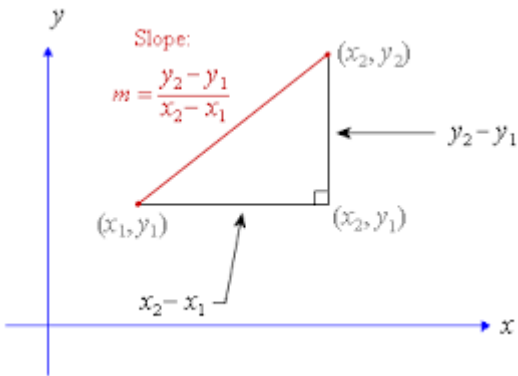
- a.) Gradient of a straight line
- b.) Equation of a straight line
- c.) The equation of a straight line of the form  $y = mx + c$
- d.) The x and y intercepts of a line
- e.) The graph of a straight line
- f.) Perpendicular lines and their gradient
- g.) Parallel lines and their gradients
- h.) Equations of parallel and perpendicular lines.

### Gradient

The steepness or slope of an area is called the gradient. Gradient is the change in y axis over the change in x axis.



$$\frac{\text{change in y co-ordinates}}{\text{change in x co-ordinates}} = \frac{y_2 - y_1}{x_2 - x_1}$$



### Note:

If an increase in the x co-ordinates also causes an increase in the y co-ordinates the gradient is positive.

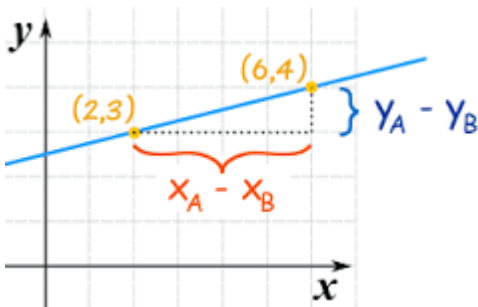
If an increase in the x co-ordinates causes a decrease in the value of the y co-ordinate, the gradient is negative.

If, for an increase in the x co-ordinate, there is no change in the value of the y co-ordinate, the gradient is zero.

For vertical line, the gradient is not defined.

### Example

Find the gradient.



Solution

$$\text{Gradient} = \frac{\text{change in y axis}}{\text{change in x axis}}$$

$$= \frac{4-3}{6-2}$$

$$= \frac{1}{4}$$

## Equation of a straight line.

Given two points

### Example.

Find the equation of the line through the points A (1, 3) and B (2, 8)

### Solution

The gradient of the required line is  $\frac{8-3}{2-1} = 5$

Take any point p (x, y) on the line. Using... points P and A, the gradient is  $\frac{y-3}{x-1}$

Therefore  $\frac{y-3}{x-1} = 5$

Hence  $y = 5x - 2$

Given the gradient and one point on the line

### Example

Determine the equation of a line with gradient 3, passing through the point (1, 5).

### Solution

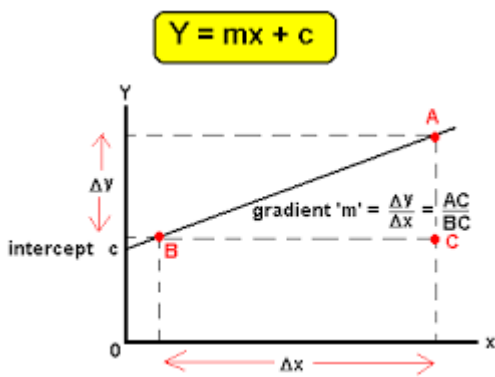
Let the line pass through a general point (x, y). The gradient of the line is  $\frac{y-5}{x-1} = 3$

Hence the equation of the line is  $y = 3x + 2$

We can express linear equation in the form  $y = mx + c$ .

## Illustrations.

For example  $4x + 3y = -8$  is equivalent to  $y = -\frac{4}{3}x - \frac{8}{3}$ . In the linear equation below gradient is equal to m while c is the y intercept.



Using the above statement we can easily get the gradient.

### Example

Find the gradient of the line whose equation is  $3y - 6x + 7 = 0$

### Solution

Write the equation in the form of  $y = mx + c$

$$3y = 6x - 7$$

$$y = 2x - \frac{7}{3}$$

$M = 2$  and also gradient is 2.

### The y- intercept

The y – intercept of a line is the value of y at the point where the line crosses the y axis. Which is C in the above figure. The x –intercept of a graph is that value of x where the graph crosses the x axis.

To find the x intercept we must find the value of y when  $x = 0$  because at every point on the y axis  $x = 0$ . The same is true for y intercept.

### Example

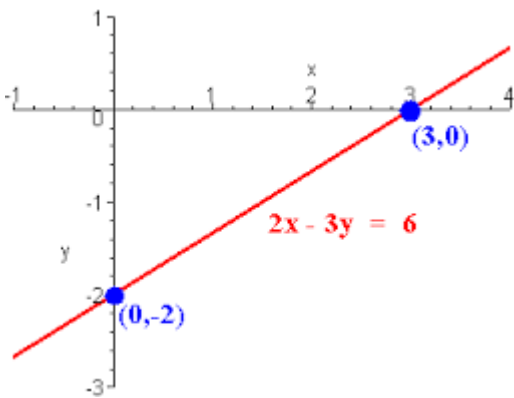
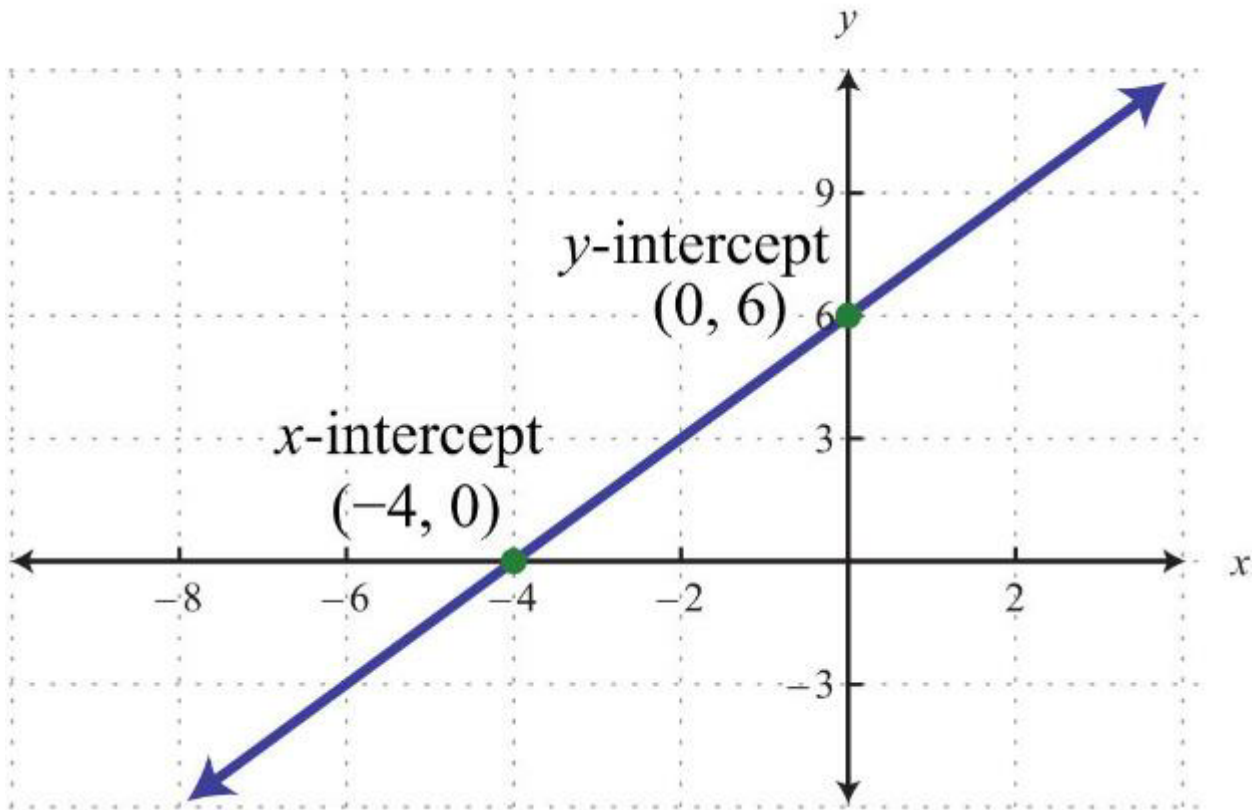
Find the y intercept  $y = 2x + 10$  on putting  $y = 0$  we have to solve this equation.

$$2x + 10 = 0$$

$$2x = -10$$

$$X = -5$$

X intercept is equal to  $-5$ .



## Perpendicular lines

If the products of the gradient of the two lines is equal to  $-1$ , then the two lines are equal to each other.

## Example

Find if the two lines are perpendicular

$$y = \frac{1}{3}x + 1 \text{ and } y = -3x - 2$$

### Solution

The gradients are

$$M = \frac{1}{3} \text{ and } M = -3$$

The product is

$$\frac{1}{3} \times -3 = -1$$

The answer is -1 hence they are perpendicular.

### Example

$$Y = 2x + 7$$

$$Y = -2x + 5$$

The products are  $2 \times -2 = -4$  hence the two lines are not perpendicular.

### Parallel lines

Parallel lines have the same gradients e.g.

$$y = 2x + 7$$

$$y = 2x - 9$$

Both lines have the same gradient which is 2 hence they are parallel

End of topic

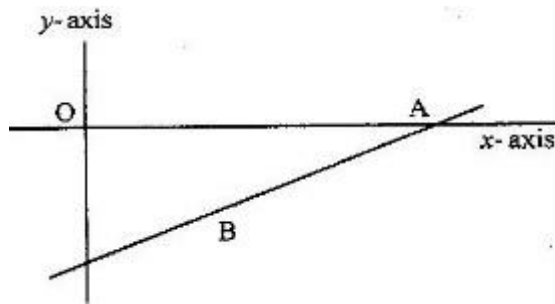
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### Past KCSE Questions on the topic

- The coordinates of the points P and Q are (1, -2) and (4, 10) respectively. A point T divides the line PQ in the ratio 2: 1
  - Determine the coordinates of T
  - (i) Find the gradient of a line perpendicular to PQ

- (ii) Hence determine the equation of the line perpendicular PQ and passing through T
- (iii) If the line meets the y-axis at R, calculate the distance TR, to three significant figures
- A line  $L_1$  passes through point (1, 2) and has a gradient of 5. Another line  $L_2$ , is perpendicular to  $L_1$  and meets it at a point where  $x = 4$ . Find the equation for  $L_2$  in the form of  $y = mx + c$
  - P (5, -4) and Q (-1, 2) are points on a straight line. Find the equation of the perpendicular bisector of PQ: giving the answer in the form  $y = mx + c$ .
  - On the diagram below, the line whose equation is  $7y - 3x + 30 = 0$  passes through the points A and B. Point A on the x-axis while point B is equidistant from x and y axes.



Calculate the co-ordinates of the points A and B

- A line with gradient of -3 passes through the points (3, k) and (k, 8). Find the value of k and hence express the equation of the line in the form  $ax + by = c$ , where a, b, and c are constants.
- Find the equation of a straight line which is equidistant from the points (2, 3) and (6, 1), expressing it in the form  $ax + by = c$  where a, b and c are constants.
- The equation of a line is  $\frac{3}{5}x + 3y = 6$ . Find the:
  - Gradient of the line (1 mk)
  - Equation of a line passing through point (1, 2) and perpendicular to the given line b
- Find the equation of the perpendicular to the line  $x + 2y = 4$  and passes through point (2, 1)
- Find the equation of the line which passes through the points P (3, 7) and Q (6, 1)
- Find the equation of the line whose x-intercept is -2 and y-intercept is 5
- Find the gradient and y-intercept of the line whose equation is  $4x - 3y - 9 = 0$

## CHAPTER TWENTY EIGHT

### REFLECTION AND CONGRUENCE

#### Specific Objectives

By the end of the topic the learner should be able to:

- a.) State the properties of reflection as a transformation
- b.) Use the properties of reflection in construction and identification of images and objects
- c.) Make geometrical deductions using reflection
- d.) Apply reflection in the Cartesian plane
- e.) Distinguish between direct and opposite congruence
- f.) Identify congruent triangles.

#### Content

- a.) Lines and planes of symmetry
- b.) Mirror lines and construction of objects and images
- c.) Reflection as a transformation
- d.) Reflection in the Cartesian plane
- e.) Direct and opposite congruency
- f.) Congruency tests (SSS, SAS, AAS, ASA and RHS)

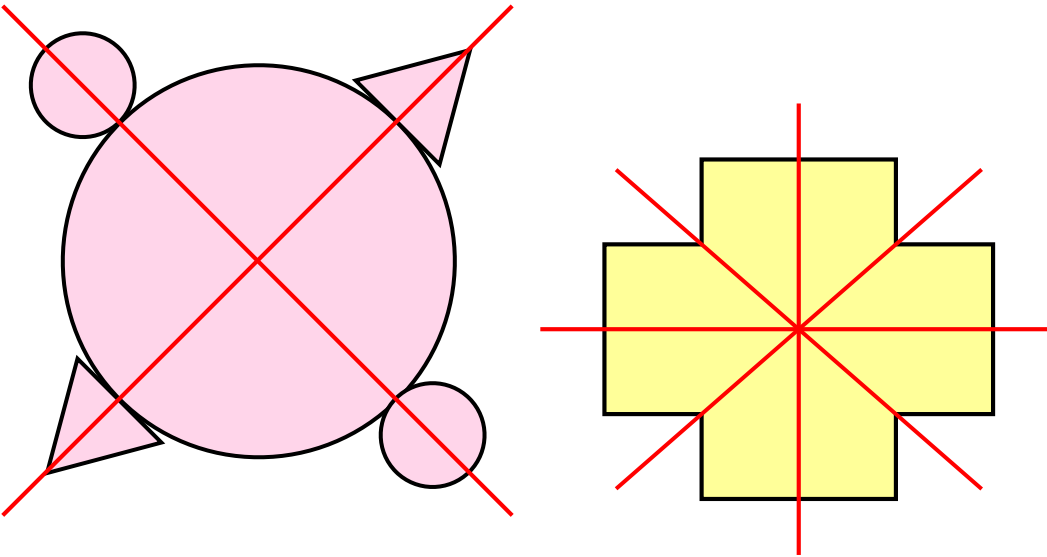
## Introduction

The process of changing the position, direction or size of a figure to form a new figure is called **transformation**.

## Reflection and congruence

### Symmetry

Symmetry is when one shape becomes exactly like another if you turn, slide or cut them into two identical parts. The lines which divides a figure into two identical parts are called lines of symmetry. If a figure is cut into two identical parts the cut part is called the plane of symmetry.



How many planes of symmetry does the above figures have?

There are two types of symmetry. Reflection and Rotational.

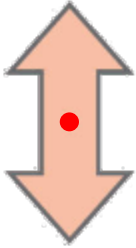
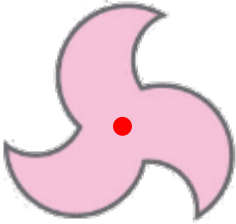
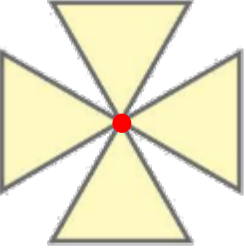

### Reflection

A transformation of a figure in which each point is replaced by a point symmetric with respect to a line or plane e.g. mirror line.

### Properties preserved under reflection

- ✓ Midpoints always remain the same.
- ✓ Angle measures remain the same i.e. the line joining a point and its image is perpendicular to the mirror line.
- ✓ A point on the object and a corresponding point on the image are equidistant from the mirror line.

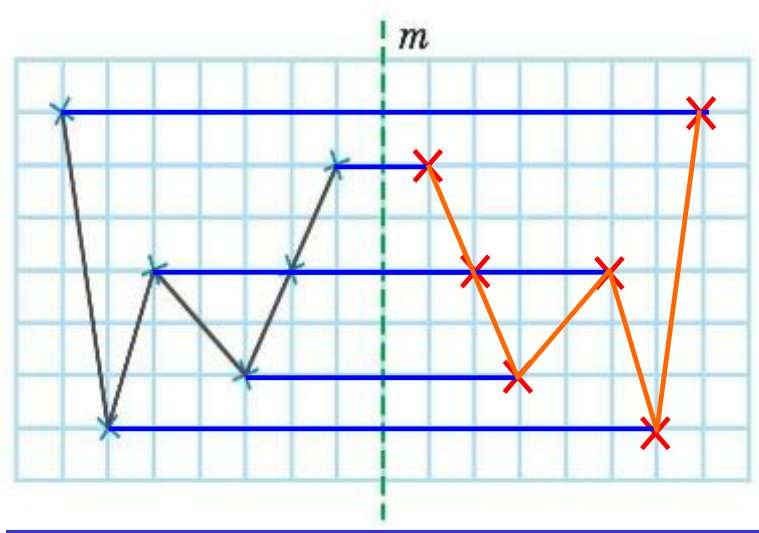
A mirror line is a line of symmetry between an object and its image.

<p>(a) <i>Figures that have rotational symmetry</i></p>				
<p>(b) <i>Order of rotational symmetry</i></p>	<p>2</p>	<p>3</p>	<p>4</p>	<p>5</p>

### Examples

To reflect an object you draw the same points of the object but on opposite side of the mirror. They must be equidistance from each other.

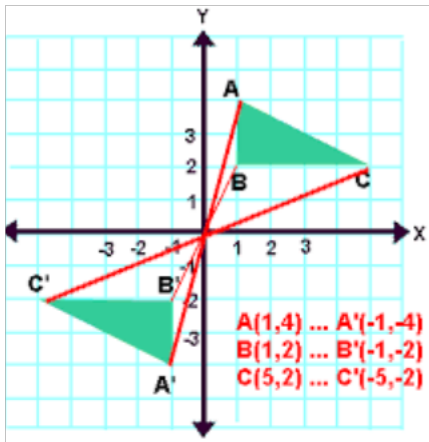
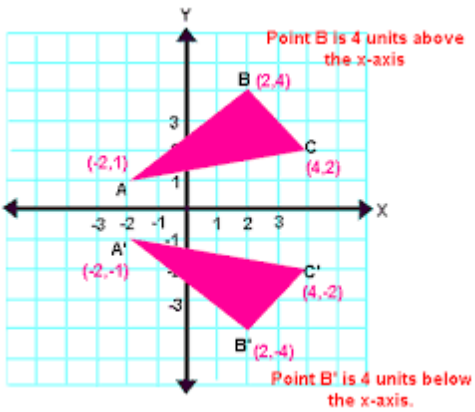
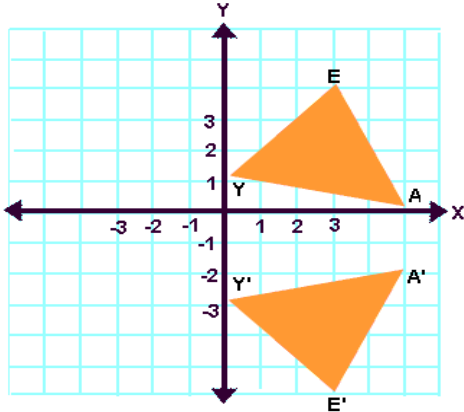
### Solution



### Exercise

Find the mirror line or the line of symmetry.

To find the mirror line, join the points on the object and image together then bisect the lines perpendicularly. The perpendicular bisector gives us the mirror line.



## Congruence

Figures with the same size and same shape are said to be congruent. If a figure fits into another directly it is said to be directly congruent.

If a figure only fits into another after it has been turned then it's called opposite congruent or indirect congruence.

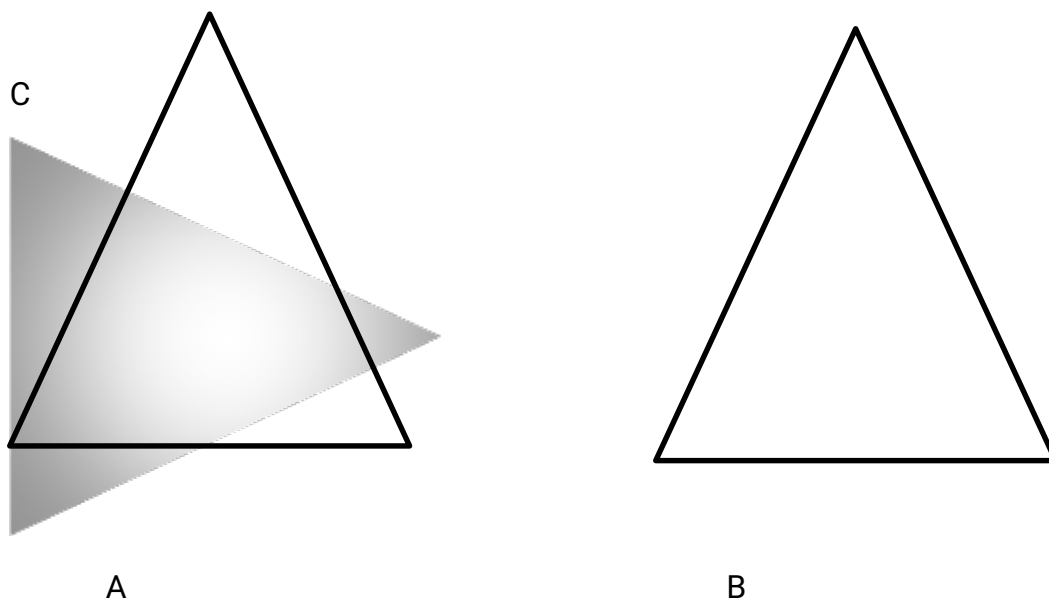


Figure A and B are directly congruent while C is oppositely or indirectly congruent because it only fits into A after it has been turned.

End of topic

Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

## CHAPTER TWENTY NINE

### ROTATION

#### Specific Objectives

By the end of the topic the learner should be able to:

- a.) State properties of rotation as a transformation
- b.) Determine centre and angle of rotation
- c.) Apply properties of rotation in the Cartesian plane
- d.) Identify point of rotational symmetry
- e.) State order of rotational symmetry of plane figure
- f.) Identify axis of rotational symmetry of solids
- g.) State order of rotational symmetry of solids
- h.) Deduce congruence from rotation.

#### Content

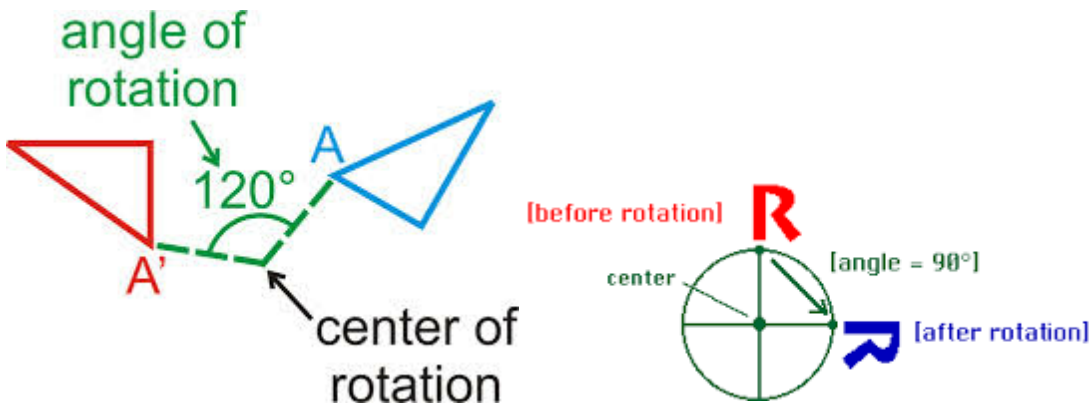
- a.) Properties of rotation
- b.) Centre and angle of rotation
- c.) Rotation in the cartesian plane
- d.) Rotational symmetry of plane figures and solids (point axis and order)
- e.) Congruence and rotation

### Introduction

A transformation in which a plane figure turns around a fixed center point called center of rotation. A rotation in the anticlockwise direction is taken to be positive whereas a rotation in the clockwise direction is taken to be negative.

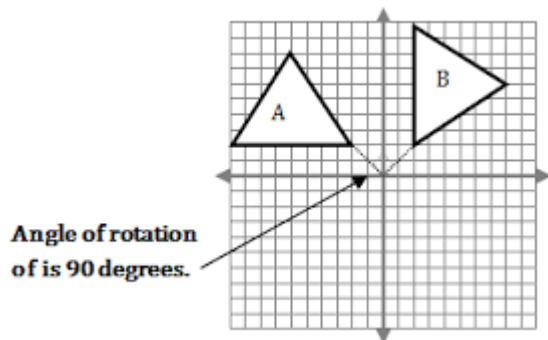
For example a rotation of  $90^{\circ}$  clockwise is taken to be negative.  $-90^{\circ}$  while a rotation of anticlockwise  $90^{\circ}$  is taken to be  $+90^{\circ}$ .

For a rotation to be completely defined the center and the angle of rotation must be stated.



### Illustration

To rotate triangle A through the origin ,angle of rotation  $+1/4$  turn.



Draw a line from each point to the center of rotation, in this case it's the origin. Measure  $90^\circ$  from the object using the protractor and make sure the base line of the protractor is on the same line as the line from the point of the object to the center. The 0 mark should start from the object.

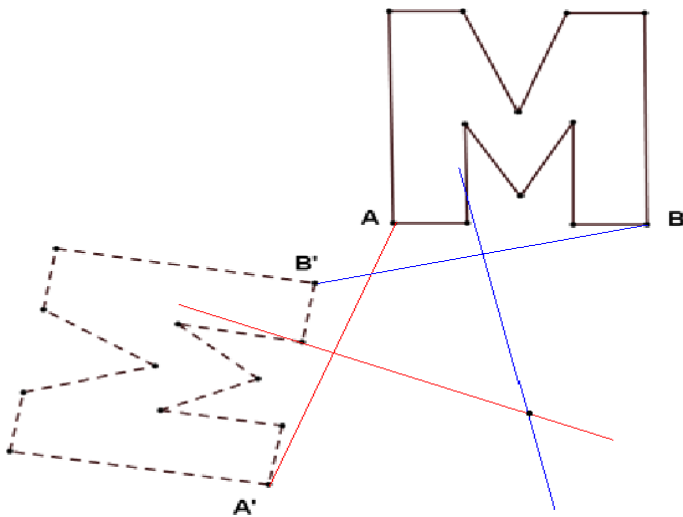
Mark  $90^\circ$  and draw a straight line to the center joining the lines at the origin. The distance from the point of the object to the center should be the same distance as the line you drew. This gives you the image point.

The distance between the object point and the image point under rotation should be the same as the center of rotation in this case  $90^\circ$ .

Illustration.

To find the center of rotation.

- ✓ Draw a segment connecting point's  $A$  and  $A'$ .
- ✓ Using a compass, find the perpendicular bisector of this line.
- ✓ Draw a segment connecting point's  $B$  and  $B'$ . Find the perpendicular bisector of this segment.
- ✓ The point of intersection of the two perpendicular bisectors is the center of rotation. Label this point  $O$ .



Justify your construction by measuring angles  $\angle AOA'$  and  $\angle BOB'$ . Did you obtain the same measure? The angle between is the angle of rotation. The zero mark of protractor should be on the object to give you the direction of rotation.

## Rotational symmetry of plane figures

The number of times the figure fits onto itself in one complete turn is called the order of rotational symmetry.

### Note;

The order of rotational symmetry of a figure =  $360 / \text{angle between two identical parts of the figure}$ .

Rotational symmetry is also called point symmetry. Rotation preserves length, angles and area, and the object and its image are directly congruent.

End of topic

Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

## CHAPTER THIRTY

### SIMILARITY AND ENLARGEMENT

#### Specific Objectives

By the end of the topic the learner should be able to:

- a.) Identify similar figures
- b.) Construct similar figures
- c.) State properties of enlargement as a transformation

- d.) Apply the properties of enlargement to construct objects and images
- e.) Apply enlargement in Cartesian planes
- f.) State the relationship between linear, area and volume scale factor
- g.) Apply the scale factors to real life situations.

### Content

- a.) Similar figures and their properties
- b.) Construction of similar figures
- c.) Properties of enlargement
- d.) Construction of objects and images under enlargement
- e.) Enlargement in the Cartesian plane
- f.) Linear, area and volume scale factors
- g.) Real life situations.

## Introduction

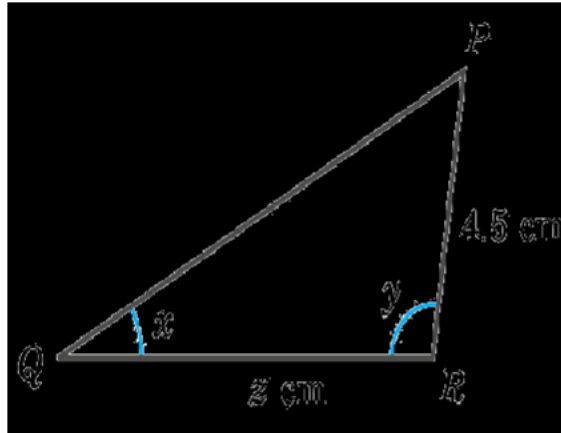
### Similar Figures

Two or more figures are said to be similar if:

- ✓ The ratio of the corresponding sides is constant.
- ✓ The corresponding angle are similar

#### Example 1

In the figures below, given that  $\triangle ABC \sim \triangle PQR$ , find the unknowns  $x$ ,  $y$  and  $z$ .



Solution

BA corresponds to QP each of them has opposite angle  $y$  and  $98^\circ$ . Hence  $y$  is equal to  $98^\circ$  BC corresponds to QR and AC corresponds to PR.

$$\frac{BA}{QR} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\frac{AC}{PR} = \frac{BC}{QR}$$

$$\frac{3}{4.5} = \frac{5}{Z}$$

$$Z = 7.5 \text{ cm}$$

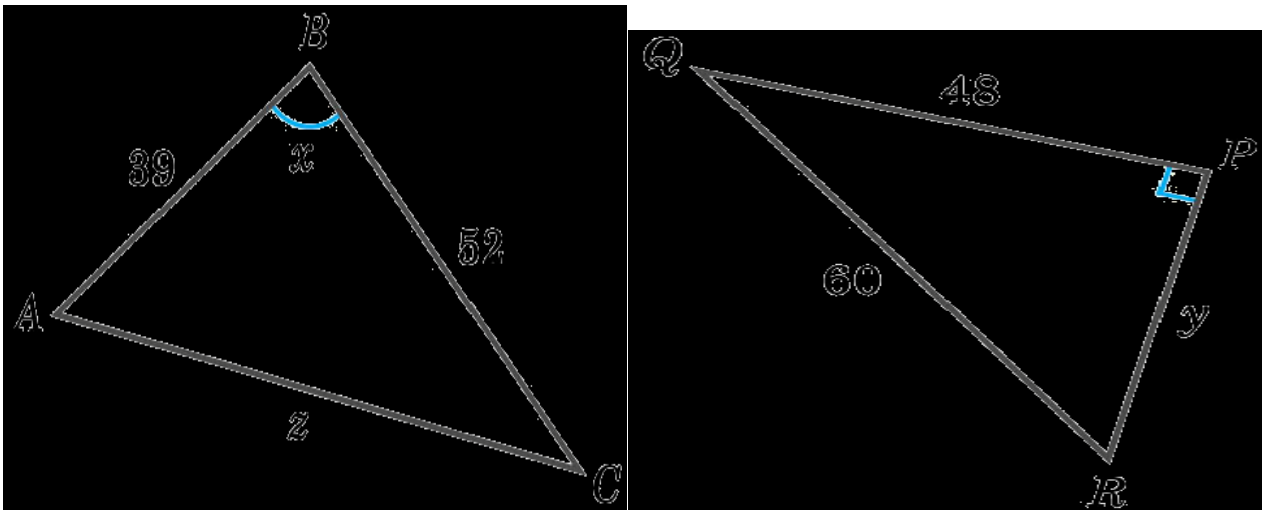
### Note:

Two figures can have the ratio of corresponding sides equal but fail to be similar if the corresponding angles are not the same.

Two triangles are similar if either their all their corresponding angles are equal or the ratio of their corresponding sides is constant.

Example:

In the figure,  $\triangle ABC$  is similar to  $\triangle RPQ$ . Find the values of the unknowns.



Since  $\triangle ABC \sim \triangle RPQ$ ,

$$\angle B = \angle P \therefore x = 90^\circ$$

Also,

$$AB/RP = BC/PQ$$

$$39/y = 52/48$$

$$(48 \times 39)$$

$$52$$

$$\therefore y = 36$$

Also,

$$AC/RQ = BC/PQ$$

$$z/60 = 52/48$$

$$\therefore z = 65$$

## ENLARGMENT

### What's enlargement?

Enlargement, sometimes called scaling, is a kind of transformation that changes the size of an

object. The image created is **similar\*** to the object. Despite the name enlargement, it includes making objects smaller.

For every enlargement, a **scale factor** must be specified. The scale factor is how many times larger than the object the image is.

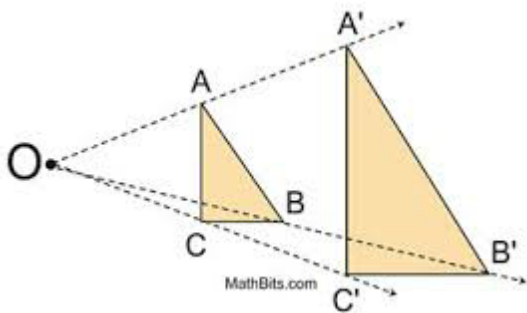
Length of side in image = length of side in object X scale factor

For any enlargement, there must be a point called the **center of enlargement**.

Distance from center of enlargement to point on image =

**Distance from Centre of enlargement to point on object X scale factor**

The Centre of enlargement can be anywhere, but it has to exist.



This process of obtaining triangle A' B 'C' from triangle A B C is called enlargement. Triangle ABC is the object and triangles A' B 'C 'Its image under enlargement scale factor 2.

Hence

$$OA'/OA=OB'/OB=OC'/OC= 2...$$

The ratio is called scale factor of enlargement. The scale factor is called liner scale factor

By measurement OA=1.5 cm, OB=3 cm and OC =2.9 cm. To get A', the image of A, we proceed as follows

$$OA=1.5 \text{ cm}$$

$$OA'/OA=2 \text{ (scale factor 2)}$$

$$OA'=1.5 \times 2$$

$$=3 \text{ cm}$$

Also  $OB'/OB=2$

$$= 3 \times 2$$

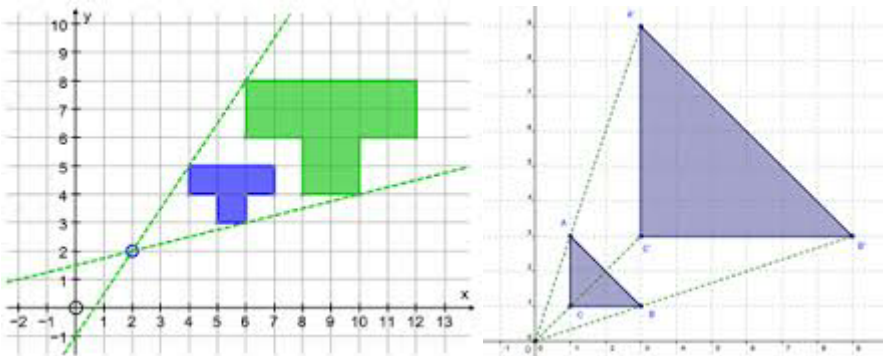
=6 cm

**Note:**

Lines joining object points to their corresponding image points meet at the Centre of enlargement.

**CENTER OF ENLARGMENT**

To find center of enlargement join object points to their corresponding image points and extend the lines, where they meet gives you the Centre of enlargement. Or Draw straight lines from each point on the image, through its corresponding point on the object, and continuing for a little further. The point where all the lines cross is the Centre of enlargement.



**SCALE FACTOR**

The scale factor can be whole number, negative or fraction. Whole number scale factor means that the image is on the same side as the object and it can be larger or the same size,

**Negative scale** factor means that the image is on the opposite side of the object and a fraction whole number scale factor means that the image is smaller either on the same side or opposite side.

**Linear scale** factor is a ratio in the form  $a : b$  or  $a/b$ . This ratio describes an enlargement or reduction in one dimension, and can be calculated using.

New length

Original length

**Area scale factor** is a ratio in the form  $e : f$  or  $e/f$ . This ratio describes how many times to enlarge. Or reduce the area of two dimensional figure. Area scale factor can be calculated using.

New Area

### Original Area

Area scale factor = (linear scale factor)<sup>2</sup>

**Volume scale factor** is the ratio that describes how many times to enlarge or reduce the volume of a three dimensional figure. Volume scale factor can be calculated using.

### New Volume

Original Volume

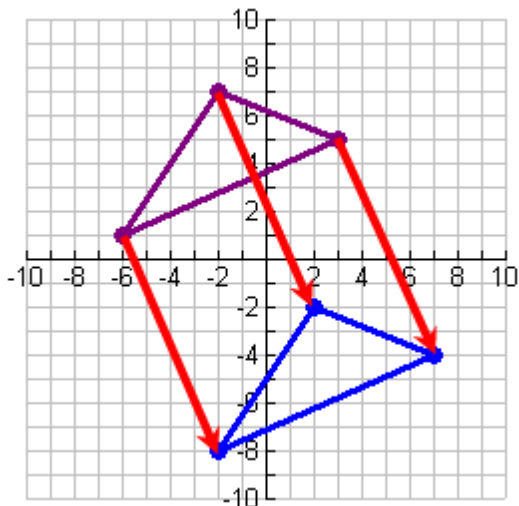
Volume scale factor = (linear scale factor)<sup>3</sup>

## CONGRUENCE TRIANGLES

When two triangles are congruent, all their **corresponding sides and corresponding angles** are equal.

## TRASLATION VECTOR

Translation vector moves every point of an object by the same amount in the given vector direction. It can be simply be defined as the addition of a constant vector to every point.



**Translations and vectors:** The translation at the left shows a vector translating the top triangle 4 units to the right and 9 units downward. The notation for such vector movement may be written as:

$$\langle 4, -9 \rangle \text{ or } \begin{pmatrix} 4 \\ -9 \end{pmatrix}$$

Vectors such as those used in translations are what is known as **free vectors**. Any two vectors of the same length and parallel to each other are considered identical. They need not have the same initial and terminal points.

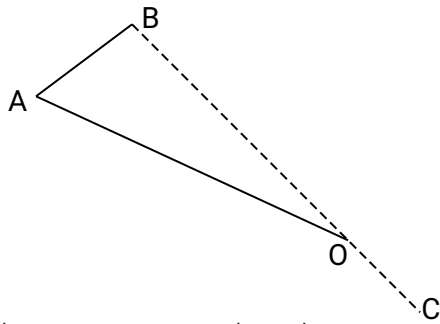
End of topic

Did you understand everything?

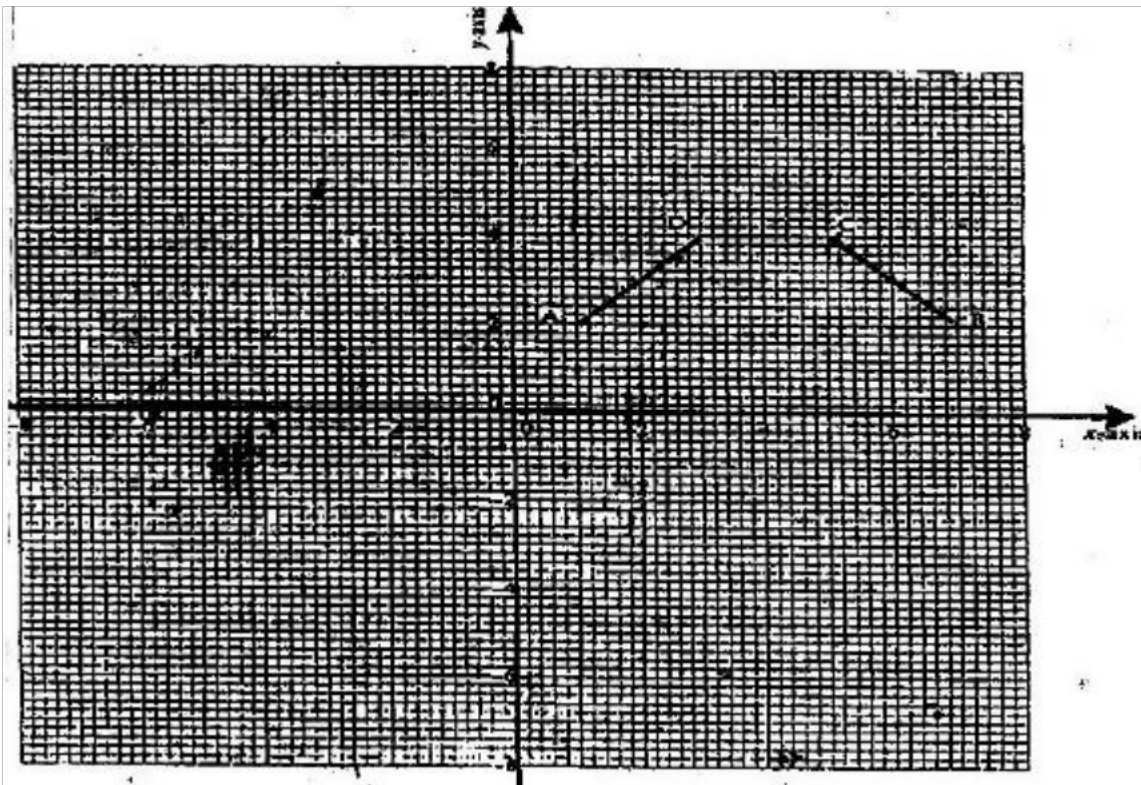
If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

## Past KCSE Questions on Reflection and Congruence, Rotation, Similarity and Enlargement.

1. A translation maps a point  $(1, 2)$  onto  $(-2, 2)$ . What would be the coordinates of the object whose image is  $(-3, -3)$  under the same translation?
2. Use binomial expression to evaluate  $(0.96)^5$  correct to 4 significant figures
11. In the figure below triangle ABO represents a part of a school badge. The badge has as symmetry of order 4 about O. Complete the figures to show the badge.



3. A point  $(-5, 4)$  is mapped onto  $(-1, -1)$  by a translation. Find the image of  $(-4, 5)$  under the same translation.
4. A triangle is formed by the coordinates A  $(2, 1)$  B  $(4, 1)$  and C  $(1, 6)$ . It is rotated clockwise through  $90^\circ$  about the origin. Find the coordinates of this image.
5. The diagram on the grid provided below shows a trapezium ABCD



On the same grid

- (a) (i) Draw the image  $A'B'C'D'$  of  $ABCD$  under a rotation of  $90^\circ$  clockwise about the origin .
- (ii) Draw the image of  $A''B''C''D''$  of  $A'B'C'D'$  under a reflection in line  $y = x$ . State coordinates of  $A''B''C''D''$ .
- (b)  $A''B''C''D''$  is the image of  $A''B''C''D''$  under the reflection in the line  $x=0$ . Draw the image  $A''B''C''D''$  and state its coordinates.
- (c) Describe a single transformation that maps  $A''B''C''D''$  onto  $ABCD$ .

6. A translation maps a point  $P(3,2)$  onto  $P'(5,4)$

(a) Determine the translation vector

(b) A point  $Q'$  is the image of the point  $Q( , 5)$  under the same translation. Find the length of  $P'Q$  leaving the answer in surd form.

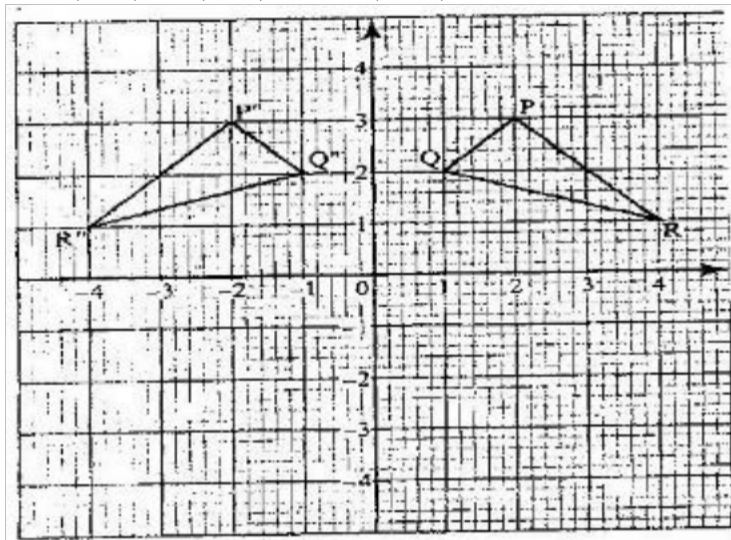
7. Two points  $P$  and  $Q$  have coordinates  $(-2, 3)$  and  $(1, 3)$  respectively. A translation maps point  $P$  to  $P'(10, 10)$

(a) Find the coordinates of  $Q'$  the image of  $Q$  under the translation (1 mk)

(b) The position vector of  $P$  and  $Q$  in (a) above are  $p$  and  $q$  respectively given that  $mp - nq = -12$

$$\begin{pmatrix} 9 \\ 9 \end{pmatrix} \text{ Find the value of } m \text{ and } n \quad (3\text{mks})$$

8. on the Cartesian plane below, triangle  $PQR$  has vertices  $P(2, 3)$ ,  $Q(1,2)$  and  $R(4,1)$  while triangle  $P''Q''R''$  has vertices  $P''(-2, 3)$ ,  $Q''(-1,2)$  and  $R''(-4, 1)$



(a) Describe fully a single transformation which maps triangle  $PQR$  onto triangle  $P''Q''R''$

(b) On the same plane, draw triangle  $P'Q'R'$ , the image of triangle  $PQR$ , under reflection in line  $y = -x$

- (c) Describe fully a single transformation which maps triangle  $P'Q'R'$  onto triangle  $P''Q''R''$
- (d) Draw triangle  $P''Q''R''$  such that it can be mapped onto triangle  $PQR$  by a positive quarter turn about  $(0, 0)$
- (e) State all pairs of triangle that are oppositely congruent

## CHAPTER THIRTY ONE

### THE PYTHAGORA'S THEOREM

#### Specific Objectives

By the end of the topic the learner should be able to:

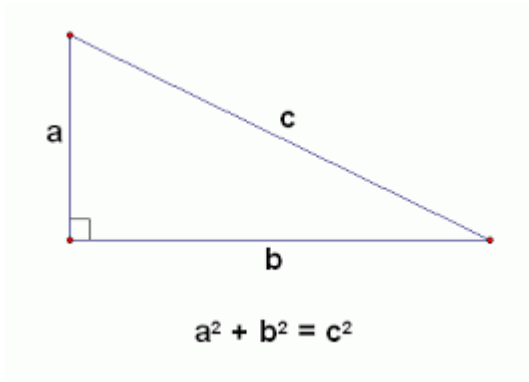
- a.) Derive Pythagoras theorem
- b.) Solve problems using Pythagoras theorem
- c.) Apply Pythagoras theorem to solve problems in life situations

#### Content

- a.) Pythagoras Theorem
- b.) Solution of problems using Pythagoras Theorem
- c.) Application to real life situations.

#### Introduction

Consider the triangle below:



Pythagoras theorem states that for a right-angled triangle, the square of the hypotenuse is equal to the sum of the square of the two shorter sides.

### Example

In a right angle triangle, the two shorter sides are 6 cm and 8 cm. Find the length of the hypotenuse.

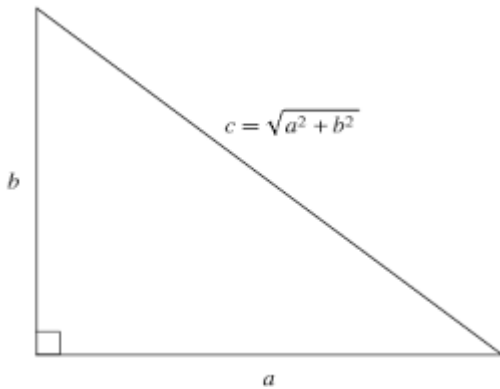
### Solution

Using Pythagoras theorem

$$\text{hyp}^2 = 6^2 + 8^2$$

$$\text{hyp}^2 = 36 + 64$$

$$\text{hyp}^2 = 100 \quad \text{hyp} = \sqrt{100} = 10$$



End of topic

Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

## Past KCSE Questions on the topic.

1. The angle of elevation of the top of a tree from a point P on the horizontal ground is  $24.5^\circ$ . From another point Q, five metres nearer to the base of the tree, the angle of elevation of the top of the tree is  $33.2^\circ$ . Calculate to one decimal place, the height of the tree.
2. A block of wood in the shape of a frustrum of a cone of slanting edge 30 cm and base radius 10cm is cut parallel to the base, one third of the way from the base along the slanting edge. Find the ratio of the volume of the cone removed to the volume of the complete cone

## CHAPTER THIRTY TWO

### TRIGONOMETRIC RATIOS

#### Specific Objectives

By the end of the topic the learner should be able to:

- a.) Define tangent, sine and cosine ratios from a right angled triangle
- b.) Read and use tables of trigonometric ratios
- c.) Use sine, cosine and tangent in calculating lengths and angles
- d.) Establish and use the relationship of sine and cosine of complimentary angles
- e.) Relate the three trigonometric ratios
- f.) Determine the trigonometric ratios of special angles  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$  and  $90^\circ$  without using tables
- g.) Read and use tables of logarithms of sine, cosine and tangent

h.) Apply the knowledge of trigonometry to real life situations.

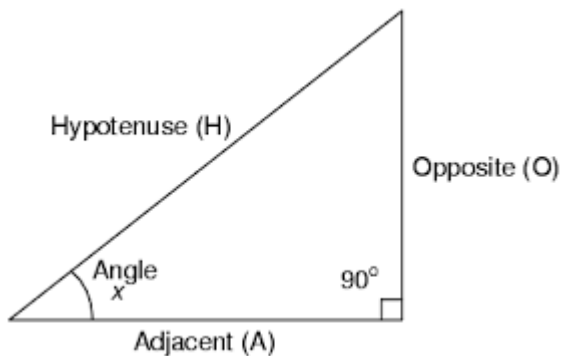
## Content

- a.) Tangent, sine and cosine of angles
- b.) Trigonometric tables
- c.) Angles and sides of a right angled triangle
- d.) Sine and cosine of complimentary angles
- e.) Relationship between tangent, sine and cosine
- f.) Trigonometric ratios of special angles  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$  and  $90^\circ$
- g.) Logarithms of sines, cosines and tangents
- h.) Application of trigonometry to real life situations.

## Introduction

### Tangent of Acute Angle

The constant ratio between the  $\frac{\text{vertical distance}}{\text{horizontal distance}}$  is called the tangent. It's abbreviated as tan



$$\text{Tan}\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

## Sine of an Angle

The ratio of the side of angle  $x$  to the hypotenuse side is called the sine.

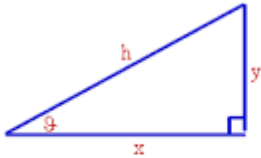
$$\text{Sin}\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

## Cosine of an Angle

The ratio of the side adjacent to the angle and hypotenuse.

$$\text{Cosine}\theta = \frac{\text{Adjacent}}{\text{hypotenuse}}$$

## Right Triangle Trigonometry



$$h = \sqrt{x^2 + y^2}$$

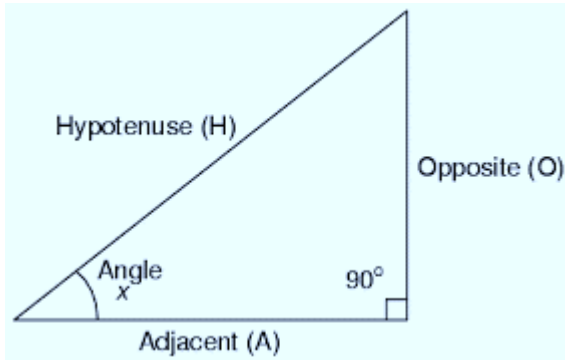
$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{h} \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{h}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x} \quad \cot \theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{x}{y}$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{h}{x} \quad \csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{h}{y}$$

$\text{sine} : \sin \theta = \frac{b}{c} = \frac{\text{side opposite } \theta}{\text{hypotenuse}}$   
 $\text{cosine} : \cos \theta = \frac{a}{c} = \frac{\text{side adjacent } \theta}{\text{hypotenuse}}$   
 $\text{tangent} : \tan \theta = \frac{b}{a} = \frac{\text{side opposite } \theta}{\text{side adjacent } \theta} = \frac{\sin \theta}{\cos \theta}$   
 $\text{cotangent} : \cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$   
 $\text{secant} : \sec \theta = \frac{1}{\cos \theta}$   
 $\text{cosecant} : \csc \theta = \frac{1}{\sin \theta}$

### Example



In the figure above adjacent length is 4 cm and Angle  $x = 36^\circ$ . Calculate the opposite length.

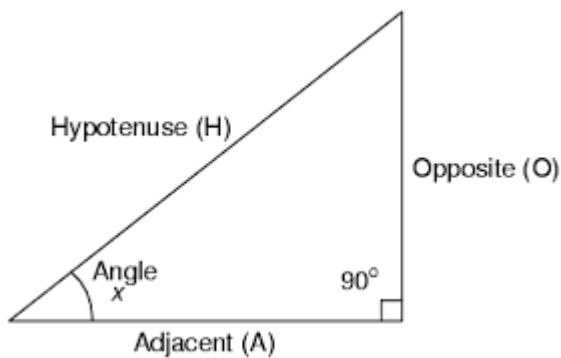
### Solution

$$\tan 36^\circ = \frac{\text{opposite length}}{\text{adjacent length}} = \frac{PR}{4}$$

$$4 \tan 36^\circ = PR$$

Therefore  $PR = 4 \times 0.7265 = 2.9060$  cm.

### Example



In the above  $o = 5$  cm  $a = 12$  cm calculate angle  $\sin x$  and cosine  $x$ .

### Solution

$$\sin x = \frac{\text{opp } o}{\text{hyp } h} = \frac{5}{h}$$

$$\begin{aligned}\text{But } H^2 &= 12^2 + 5^2 \\ &= 169 \\ &= \sqrt{169} \\ H &= 13\end{aligned}$$

$$\begin{aligned}\text{Therefore } \sin x &= \frac{5}{13} \\ &= 0.3846 \\ \cos x &= \frac{\text{adj}}{\text{hyp}} \\ &= \frac{12}{13} \\ &= 0.9231\end{aligned}$$

### Sine and cosines of complementary angles

For any two complementary angles  $x$  and  $y$ ,  $\sin x = \cos y$   $\cos x = \sin y$  e.g.  $\sin 60^\circ = \cos 30^\circ$ ,

$$\sin 30^\circ = \cos 60^\circ, \sin 70^\circ = \cos 20^\circ,$$

### Example

Find acute angles  $\alpha$  and  $\beta$  if

$$\sin \alpha = \cos 33^\circ$$

### Solution

$$\sin \alpha = \cos 33$$

$$\text{Therefore } \alpha + 33 = 90$$

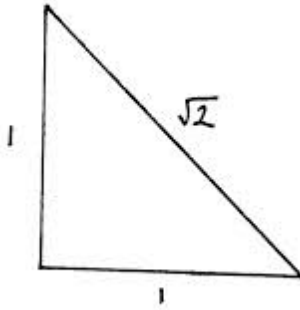
$$\alpha = 57^\circ$$

### Trigonometric ratios of special Angles $30^\circ$ $45^\circ$ $60^\circ$ .

These trigonometric ratios can be deduced by the use of isosceles right – angled triangle and equilateral triangles as follows.

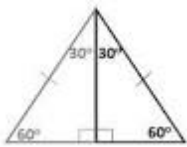
### Tangent cosine and sine of $45^\circ$ .

The triangle should have a base and a height of one unit each, giving hypotenuse of  $\sqrt{2}$ .



$$\cos 45^\circ = \frac{1}{\sqrt{2}} \quad \sin 45^\circ = \frac{1}{\sqrt{2}} \quad \tan 45^\circ = 1$$

Tangent cosine and sine of  $30^\circ$  and  $60^\circ$ .



The equilateral triangle has a sides of 2 units each

$$\sin 30^\circ = \frac{1}{2} \quad \cos 30^\circ = \frac{\sqrt{3}}{2} \quad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \cos 60^\circ = \frac{1}{2} \quad \tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

End of topic

Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

## Past KCSE Questions on the topic.

- Given  $\sin (90 - a) = \frac{1}{2}$ , find without using trigonometric tables the value of  $\cos a$  (2mks)
- If  $\tan \theta = \frac{24}{45}$ , find without using tables or calculator, the value of

$$\frac{\tan \theta - \cos \theta}{\cos \theta + \sin \theta}$$

(3 marks)

3. At point A, David observed the top of a tall building at an angle of  $30^\circ$ . After walking for 100 meters towards the foot of the building he stopped at point B where he observed it again at an angle of  $60^\circ$ . Find the height of the building
4. Find the value of  $\theta$ , given that  $\frac{1}{2} \sin \theta = 0.35$  for  $0^\circ \leq \theta \leq 360^\circ$
5. A man walks from point A towards the foot of a tall building 240 m away. After covering 180m, he observes that the angle of elevation of the top of the building is  $45^\circ$ . Determine the angle of elevation of the top of the building from A
6. Solve for x in  $2 \cos 2x = 0.6000$   $0^\circ \leq x \leq 360^\circ$ .
7. Wangechi whose eye level is 182cm tall observed the angle of elevation to the top of her house to be  $32^\circ$  from her eye level at point A. she walks 20m towards the house on a straight line to a point B at which point she observes the angle of elevation to the top of the building to be  $40^\circ$ . Calculate, correct to 2 decimal places the ;
- distance of A from the house
  - The height of the house
8. Given that  $\cos A = \frac{5}{13}$  and angle A is acute, find the value of:-  
 $2 \tan A + 3 \sin A$
9. Given that  $\tan 5^\circ = 3 + 5\sqrt{5}$ , without using tables or a calculator, determine  $\tan 25^\circ$ , leaving your answer in the form  $a + b\sqrt{c}$
10. Given that  $\tan x = \frac{5}{12}$ , find the value of the following without using mathematical tables or calculator:
- $\cos x$
  - $\sin^2(90-x)$
11. If  $\tan \theta = \frac{8}{15}$ , find the value of  $\sin \theta - \cos \theta$  without using a calculator or table  
 **$\cos \theta + \sin \theta$**

## CHAPTER THIRTY THREE

### AREA OF A TRIANGLE

#### Specific Objectives

By the end of the topic the learner should be able to:

- Derive the formula; Area =  $\frac{1}{2}ab \sin C$
- Solve problems involving area of triangles using the formula Area =  $\frac{1}{2}ab \sin C$ ;
- Solve problems on area of a triangle using the formula area =  $\sqrt{s(s-a)(s-b)(s-c)}$

#### Content

- Area of triangle A =  $\frac{1}{2} ab \sin C$
- Area of a triangle A =  $\sqrt{s(s-a)(s-b)(s-c)}$
- Application of the above formulae in solving problems involving real life situations.

## Introduction

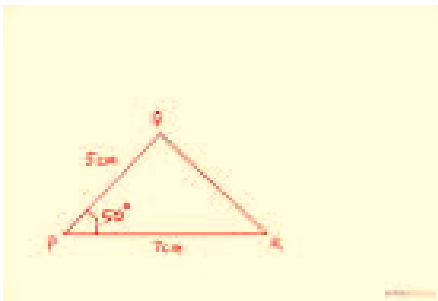
### Area of a triangle given two sides and an included Angle

The area of a triangle is given by  $A = \frac{1}{2}bh$  but sometimes we use other formulas to as follows.

### Example

If the length of two sides and an included angle of a triangle are given, the area of the triangle is

given by  $A = \frac{1}{2}absin\theta$



In the figure above PQ is 5 cm and PR is 7 cm angle QPR is  $50^\circ$ . Find the area of the the triangle.

### Solution

Using the formulae by  $A = \frac{1}{2}absin\theta$   $a = 5$  cm  $b = 7$  cm and  $\theta = 50^\circ$

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times 5 \times 7 \sin 50^\circ \\ &= 2.5 \times 7 \times 0.7660 \\ &= 13.40 \text{ cm}^2\end{aligned}$$

### Area of the triangle, given the three sides.

#### Example

Find the area of a triangle ABC in which AB = 5 cm, BC = 6 cm and AC = 7 cm.

#### Solution

When only three sides are given us the formulae

$$A = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{Hero's formulae}$$

$$S = \frac{1}{2} \text{ of the perimeter of the triangle}$$

$$= \frac{1}{2}(a + b + c) \quad A, b, c \text{ are the lengths of the sides of the triangle.}$$

$$\begin{aligned} &= \frac{1}{2}(6+7+5) = 9 \quad \text{And } A = \sqrt{9(9-6)(9-7)(9-5)} \\ &= \sqrt{9 \times 3 \times 2 \times 4} \\ &= \sqrt{216} \\ &= 14.70 \text{ cm}^2\end{aligned}$$

End of topic

Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

### Past KCSE Questions on the topic.

1. The sides of a triangle are in the ratio 3:5:6. If its perimeter is 56 cm, use the Hero's formula to find its area (4mks)
2. The figure below is a triangle XYZ. ZY = 13.4cm, XY = 5cm and angle xyz = 57.7°



Calculate

- i.) Length XZ. (3mks)
- i.) Angle XZY. (2 mks)
- ii.) If a perpendicular is dropped from point X to cut ZY at M, Find the ratio MY: ZM. (3 mks)

Find the area of triangle XYZ. (2 mks)

## CHAPTER THIRTY FOUR

### AREA OF QUADRILATERALS

#### Specific Objectives

By the end of the topic the learner should be able to:

- a.) Find the area of a quadrilateral
- b.) Find the area of other polygons (regular and irregular).

#### Content

- a.) Area of quadrilaterals
- b.) Area of other polygons (regular and irregular).

## Introduction

### Quadrilaterals.

They are four sided figures e.g. rectangle, square, rhombus, parallelogram, trapezium and kite.

### Area of rectangle

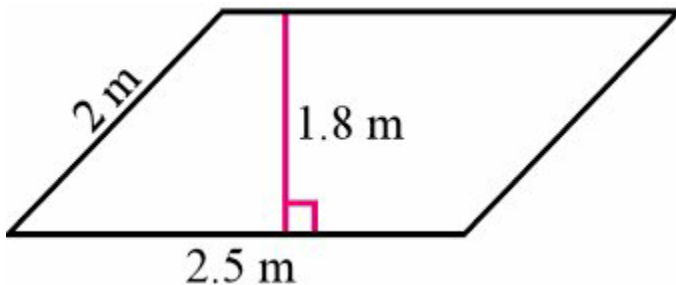
$$A = L \times W$$



AB and DC are the lengths while AD and BC are the width.

### Area of parallelogram

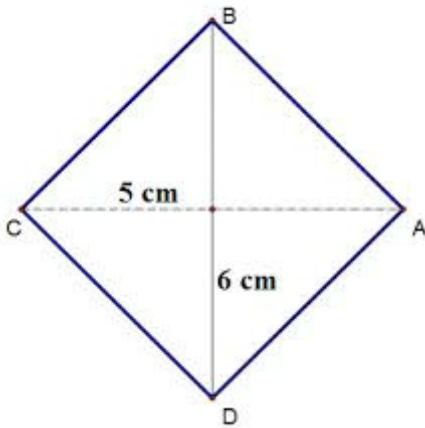
A figure whose opposite sides are equal and parallel.



$$\text{Area} = \text{base} \times \text{height} = 2.5 \times 1.8 = 4.5 \text{ cm}^2$$

### Area of a Rhombus.

A figure with all sides equal and the diagonals bisect each other at  $90^\circ$ . In the figure below  $BC = CD = DA = AB = 4 \text{ cm}$  while  $AC = 10 \text{ cm}$  and  $BD = 12$ . Find the area



### Solution

Find half of the diagonal which is  $\frac{1}{2} \times 10 = 5 \text{ cm}$

$$\text{Area of } \Delta BCD = \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$$

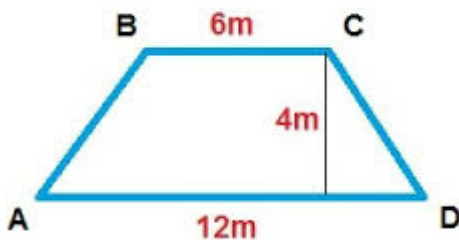
Area of  $\Delta ABCD = 2 \times \text{area of } \Delta BCD$

$$= 2 \times 30 \text{ cm}^2$$

$$= 60 \text{ cm}^2$$

### Area of Trapezium

A quadrilateral with only two of its opposite sides being parallel. The area =  $\left(\frac{a+b}{2}\right)h$



### Example

Find the area of the above figure

Solution

$$\text{Area} = \left(\frac{6+12}{2}\right)4$$

$$= 9 \times 4 = 36 \text{ cm}^2$$

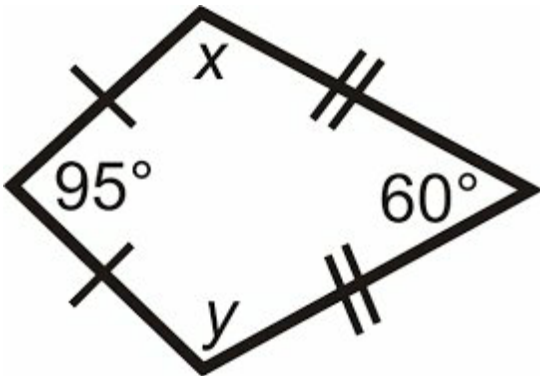
Note:

You can use the sine rule to get the height given the hypotenuse and an angle.

$$\sin\theta = \frac{\text{opposite}}{\text{hypoteneus}}$$

Or use the acronym SOHCAHTOA

Rhombus



Example

In the figure above the lines marked // = 7 cm while / = 5 cm, find the area.

Solution

Join X to Y.

Find the area of the two triangles formed

$$\frac{1}{2} \times 5 \times 5 \times \sin 95^\circ = 12.45 \text{ (Triangle one)}$$

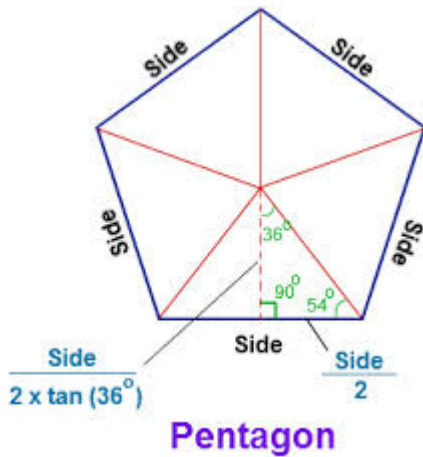
$$\frac{1}{2} \times 7 \times 7 \times \sin 60^\circ = 21.21 \text{ (Triangle two)}$$

Then add the area of the two triangles

$$12.45 + 21.21 = 33.67 \text{ cm}^2$$

Area of regular polygons

Any regular polygon can be divided into isosceles triangles by joining the vertices to the Centre. The number of the polygon formed is equal to the number of sides of the polygon.



### Example

If the radius is of a pentagon 6 cm find its area.

### Solution

Divide the pentagon into five triangles each with  $72^\circ$  ie  $(\frac{360}{5})$

$$\begin{aligned} \text{Area of one triangle will be} &= \frac{1}{2} \times 6 \times 6 \times \sin 72^\circ \\ &= 17.11 \end{aligned}$$

There are five triangles therefore

$$\begin{aligned} \text{AREA} &= 5 \times 17.11 \\ &= 85.55 \text{cm}^2 \end{aligned}$$

End of topic

Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

## Past KCSE Questions on the topic.

1.) The diagram below, not drawn to scale, is a regular pentagon circumscribed in a circle of radius 10 cm at centre O

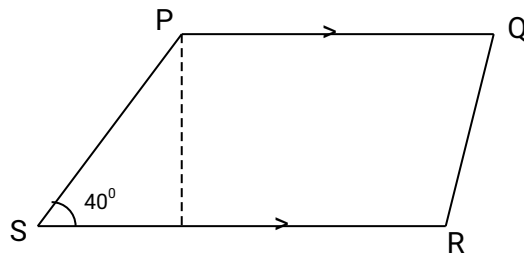


Find

(a) The side of the pentagon (2mks)

(b) The area of the shaded region (3mks)

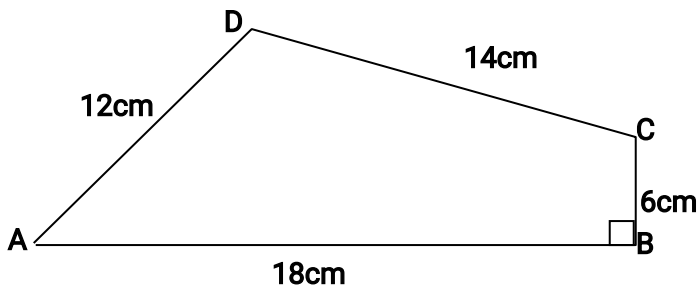
2.) PQRS is a trapezium in which PQ is parallel to SR, PQ = 6cm, SR = 12cm,  $\angle PSR = 40^\circ$  and PS = 10cm. Calculate the area of the trapezium. (4mks)



3.) A regular octagon has an area of  $101.8 \text{ cm}^2$ . calculate the length of one side of the octagon (4marks)

4.) Find the area of a regular polygon of length 10 cm and side  $n$ , given that the sum of interior angles of  $n$  :  $n - 1$  is in the ratio 4 : 3.

1.) Calculate the area of the quadrilateral ABCD shown:-



## CHAPTER THIRTY FIVE

### AREA PART OF A CIRCLE

#### Specific Objectives

By the end of the topic the learner should be able to:

- a.) Find the area of a sector
- b.) Find the area of a segment
- c.) Find the area of a common region between two circles.

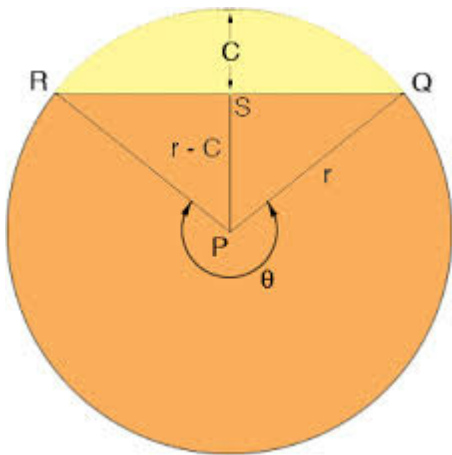
#### Content

- a.) Area of a sector
- b.) Area of a segment
- c.) Area of common regions between circles.

## Introduction

### Sector

A sector is an area bounded by two radii and an arc .A minor sector has a smaller area compared to a major sector.



The orange part is the major sector while the yellow part is the minor sector.

### The area of a sector

The area of a sector subtending an angle  $\theta$  at the Centre of the circle is given by;  $A = \frac{\theta}{360} \times \pi r^2$

### Example

Find the area of a sector of radius 3 cm, if the angle subtended at the Centre is given as  $140^\circ$  take

$$\pi \text{ as } \frac{22}{7}$$

### Solution

Area A of a sector is given by;

$$A = \frac{\theta}{360} \times \pi r^2$$

$$\begin{aligned} \text{Area} &= \frac{140}{360} \times \frac{22}{7} \times 3^2 \\ &= 11 \text{ cm}^2 \end{aligned}$$

### Example

The area of the sector of a circle is 38.5 cm. Find the radius of the circle if the angle subtended at the Centre is  $90^\circ$ .

### Solution

From  $A = \frac{\theta}{360} \times \pi r^2$ , we get

$$\frac{90}{360} \times \frac{22}{7} \times r^2 = 38.5$$

$$r^2 = \frac{38.5 \times 360 \times 7}{90 \times 22}$$

$$r^2 = \sqrt{49}$$

$$R = 7 \text{ cm}$$

### Example

The area of a sector of radius 63 cm is 4158 cm. Calculate the angle subtended at the Centre of the circle.

### Solution

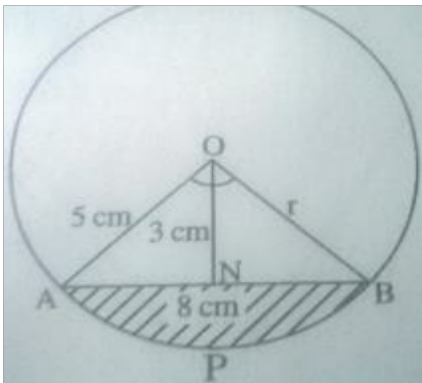
$$4158 = \frac{\theta}{360} \times \frac{22}{7} \times 63 \times 63$$

$$\theta = \frac{4158 \times 360 \times 7}{22 \times 63 \times 63}$$

$$= 120^\circ$$

### Area of a segment of a circle

A segment is a region of a circle bounded by a chord and an arc.



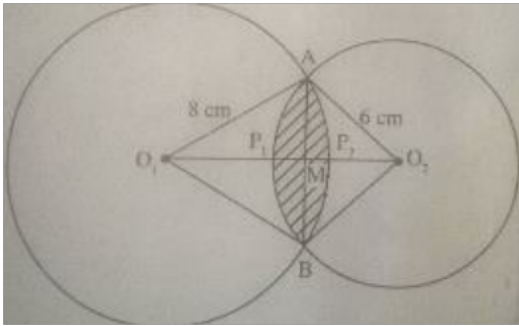
In the figure above the shaded region is a segment of the circle with Centre O and radius r. AB=8 cm, ON = 3 cm, ANGLE AOB =106.3<sup>0</sup>. Find the area of the shaded part.

### Solution

Area of the segment = area of the sector OAPB – area of triangle OAB

$$\begin{aligned} &= \left[ \frac{106.3}{360} \times 3.142 \times 5^2 \right] - \left[ \frac{1}{2} \times 8 \times 3 \right] \\ &= 23.19 - 12 \\ &= 11.19 \text{ cm}^2 \end{aligned}$$

Area of a common region between two intersecting circles.



Find the area of the intersecting circles above. If the common chord AB is 9 cm.

### Solution

From  $\triangle AO_1M$ ;

$$\begin{aligned} O_1M &= \sqrt{8^2 - 4.5^2} \\ &= \sqrt{43.75} \end{aligned}$$

=6.614 cm

From  $\triangle AO_2M$ ;

$$\begin{aligned} O_2M &= \sqrt{6^2 - 4.5^2} \\ &= \sqrt{15.75} \end{aligned}$$

=3.969 cm

The area between the intersecting circles is the sum of the areas of segments  $AP_1B$  and  $AP_2B$ .  
Area of segment  $AP_1B$  = area of sector  $O_2AP_1B$  - area of  $\triangle O_2AB$

Using trigonometry,  $\sin \angle AO_2M = \frac{AM}{AO_2} = \frac{4.5}{6} = 0.75$

Find the sine inverse of 0.75 to get  $48.59^\circ$  hence  $\angle AO_2M = 48.59^\circ$

$$\angle AO_2B = 2 \times \angle AO_2M$$

$$= 2 \times 48.59^\circ = 97.18^\circ$$

$$\text{Area of segment } AP_1B = \frac{97.18}{360} \times 3.14 \times 6^2 - \frac{1}{2} \times 9 \times 3.969$$

$$= 30.53 - 17.86$$

$$= 12.67 \text{ cm}^2$$

Area of segment  $AP_2B$  = area of sector  $O_1AP_2B$  - area of  $\Delta O_1AB$

Using trigonometry,  $\sin \angle AO_1M = \frac{AM}{AO_1} = \frac{4.5}{8} = 0.5625$

Find the sine inverse of 0.5625 to get  $34.23^\circ$  hence  $\angle AO_1M = 34.23^\circ$

$$\begin{aligned} \angle AO_1B &= 2 \times \angle AO_1M \\ &= 2 \times 34.23^\circ \\ &= 68.46^\circ \end{aligned}$$

$$\begin{aligned} \text{Area of segment } AP_2B &= \frac{68.46}{360} \times 3.12 \times 8^2 - \frac{1}{2} \times 9 \times 6.614 \\ &= 38.24 - 29.76 \\ &= 8.48 \text{ cm}^2 \end{aligned}$$

Therefore the area of the region between the intersecting circles is given by;

$$\begin{aligned} &\text{Area of segment } AP_1B + \text{area of segment } AP_2B \\ &= 12.67 + 8.48 \\ &= 21.15 \text{ cm}^2 \end{aligned}$$

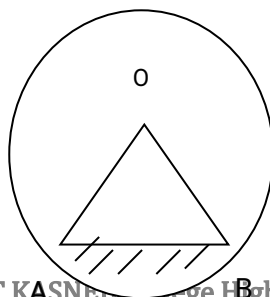
End of topic

Did you understand everything?

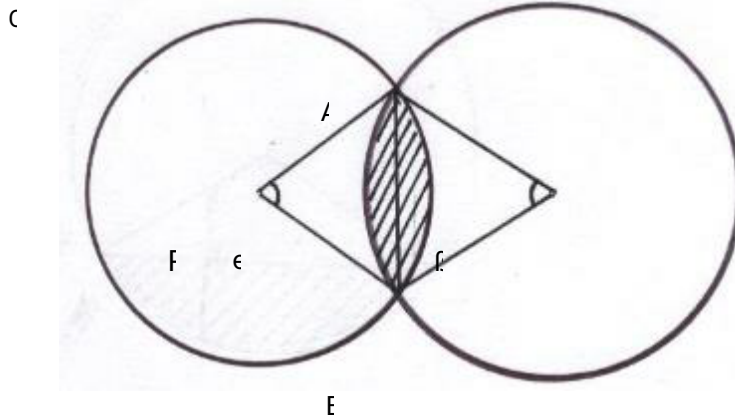
If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

## Past KCSE Questions on the topic.

- The figure below shows a circle of radius 9cm and centre O. Chord AB is 7cm long. Calculate the area of the shaded region. (4mks)



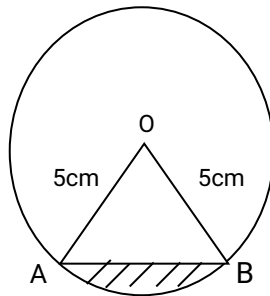
2. The figure below shows two intersecting circles with centres P and Q of radius 8cm and 10cm respectively. Length AB = 12cm



Calculate:

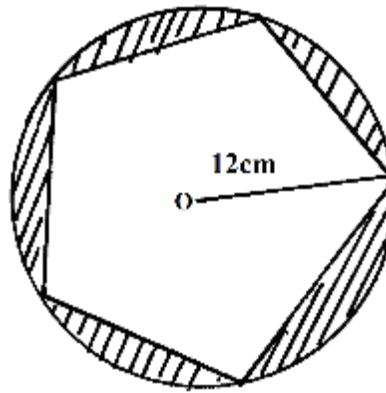
- a)  $\angle APB$  (2mks)
- b)  $\angle AQB$  (2mks)
- c) Area of the shaded region (6mks)

- 3.



The diagram above represents a circle centre o of radius 5cm. The minor arc AB subtends an angle of  $120^\circ$  at the centre. Find the area of the shaded part. (3mks)

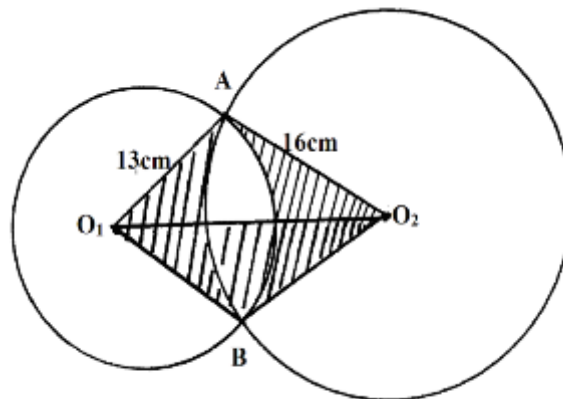
4. The figure below shows a regular pentagon inscribed in a circle of radius 12cm, centre O.



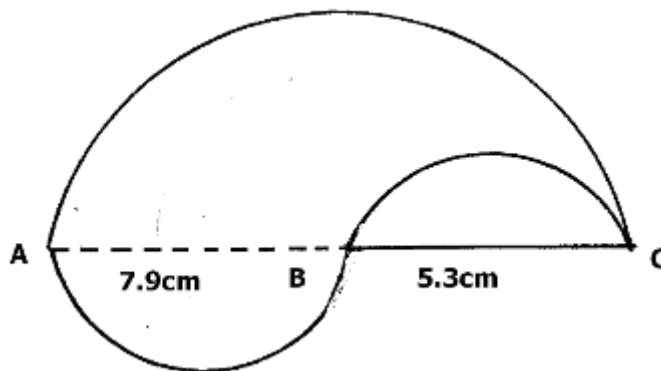
Calculate the area of the shaded part.

(3mks)

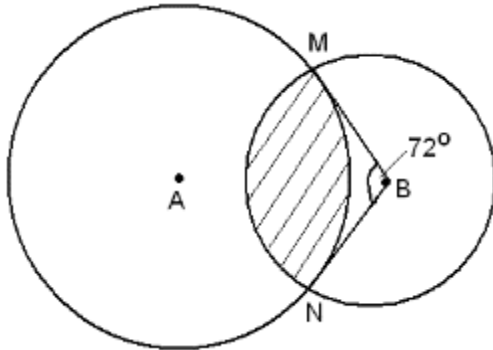
5. Two circles of radii 13cm and 16cm intersect such that they share a common chord of length 20cm. Calculate the area of the shaded part.  $\left(\pi = \frac{22}{7}\right)$  (10mks)



6. Find the perimeter of the figure below, given AB, BC and AC are diameters. (4mks)



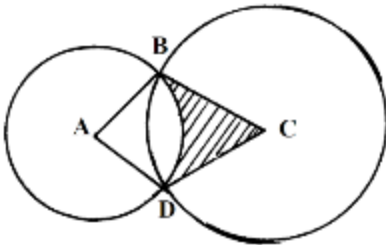
7. The figure below shows two intersecting circles. The radius of a circle A is 12cm and that of circle B is 8 cm.



If the angle  $MBN = 72^\circ$ , calculate

The size of the angle  $MAN$

- b) The length of  $MN$   
 c) The area of the shaded region.
- 8.



In the diagram above, two circles, centres A and C and radii 7cm and 24cm respectively intersect at B and D.  $AC = 25\text{cm}$ .

a) Show that angle  $ABC = 90^\circ$

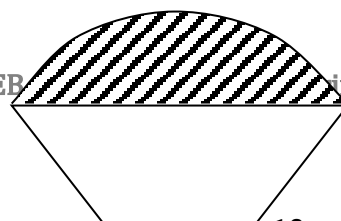
b) Calculate

i) the size of obtuse angle  $BAD$

ii) the area of the shaded part

(10 Mks)

9. The ends of the roof of a workshop are segments of a circle of radius 10m. The roof is 20m long. The angle at the centre of the circle is  $120^\circ$  as shown in the figure below:

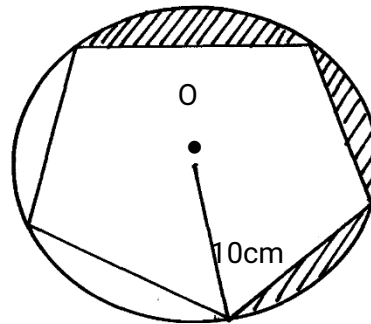


(a) **Calculate** :-

- (i) The area of one end of the roof
- (ii) The area of the curved surface of the roof

(b) What would be the cost to the nearest shilling of covering the two ends and the curved surface with galvanized iron sheets costing shs.310 per square metre

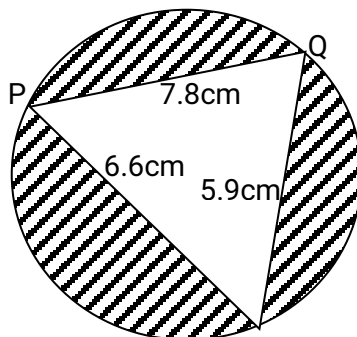
10. The diagram below, not drawn to scale, is a regular pentagon circumscribed in a circle of radius 10cm at centre O



Find;

- (a) The side of the pentagon
- (b) The area of the shaded region

11. Triangle **PQR** is inscribed in the circle **PQ**= 7.8cm, **PR** = 6.6cm and **QR** = 5.9cm. Find:



- (a) The radius of the circle, correct to one decimal place
- (b) The angles of the triangle
- (c) The area of shaded region

## CHAPTER THIRTY SIX

### SURFACE AREA OF SOLIDS

#### Specific Objectives

By the end of the topic the learner should be able to:

- a.) Find the surface area of a prism
- b.) Find the surface area of a pyramid
- c.) Find the surface area of a cone
- d.) Find the surface area of a frustum
- e.) Find the surface area of a sphere and a hemisphere.

#### Content

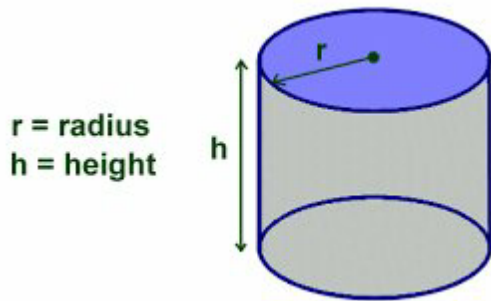
Surface area of prisms, pyramids, cones, frustums and spheres.

## Introduction

### Surface area of a prism

A prism is a solid with uniform cross- section. The surface area of a prism is the sum of its faces.

### Cylinder



Area of closed cylinder =  $2\pi r^2 + 2\pi r l$

Area of open cylinder =  $\pi r^2 + 2\pi r l$  ( area of the bottom circle as the top is open)

### Example

Find the area of the closed cylinder  $r = 2.8$  cm and  $l = 13$  cm

Solution

$$\begin{aligned} &= 2 \left( \frac{22}{7} \times 2.8 \times 2.8 \right) + \left( 2 \times \frac{22}{7} \times 2.8 \times 13 \right) \\ &= 49.28 \text{ cm}^2 + 228.8^2 \\ &= 278.08 \text{ cm}^2 \end{aligned}$$

Note;

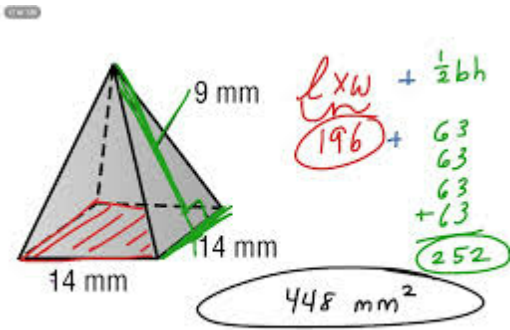
For open cylinder do not multiply by two, find the area of only one circle.

### Surface area of a pyramid

The surface area of a pyramid is the sum of the area of the slanting faces and the area of the base.

Surface area = base area + area of the four triangular faces (take the slanting height marked green below)

### Example



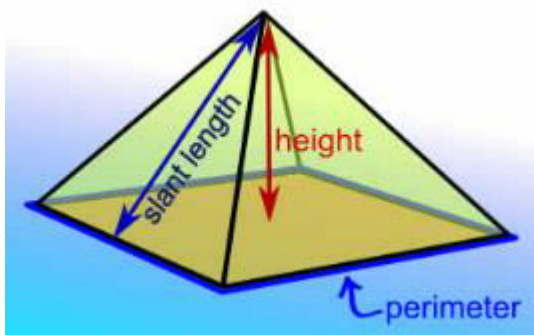
**Solution**

Surface area = base area + area of the four triangular faces

$$= (14 \times 14) + \left(\frac{1}{2} \times 14 \times 14\right)$$

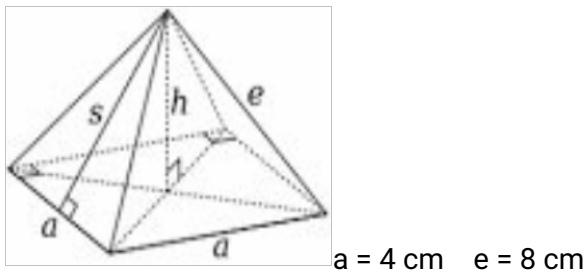
$$= 196 + 252$$

$$= 448 \text{ mm}^2$$



**Example**

The figure below is a right pyramid with a square base of 4 cm and a slanting edge of 8 cm. Find the surface area of the pyramid.



Surface area = base area + area of the four triangular bases

$$= (l \times w) + 4 \left(\frac{1}{2}bh\right)$$

Remember height is the slanting height

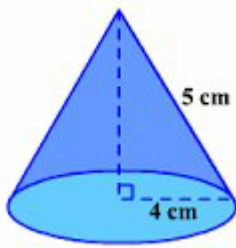
$$\begin{aligned} \text{Slanting height} &= \sqrt{8^2 - 2^2} \\ &= \sqrt{60} \end{aligned}$$

$$\begin{aligned} \text{Surface area} &= (4 \times 4) + 4\left(\frac{1}{2} \times 4 \times \sqrt{60}\right) \\ &= 77.97 \text{ cm}^2 \end{aligned}$$

### Surface area of a cone

$$\text{Total surface area of a cone} = \pi r^2 + \pi r l$$

$$\text{Curved surface area of a cone} = \pi r l$$



### Example

Find the surface area of the cone above

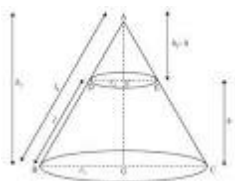
$$\begin{aligned} &= (3.14 \times 4 \times 4) + (3.14 \times 4 \times 5) \\ &= 50.24 + 62.8 \\ &= 113.04 \text{ cm}^2 \end{aligned}$$

### Note;

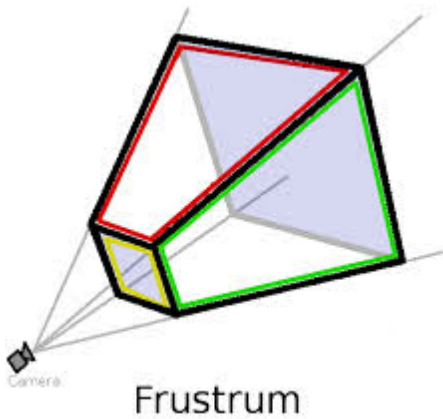
Always use slanting height, if it's not given find it using Pythagoras theorem

### Surface area of a frustum

The bottom part of a cut pyramid or cone is called a frustum. Example of frustums are bucket,



Examples a lampshade and a hopper.

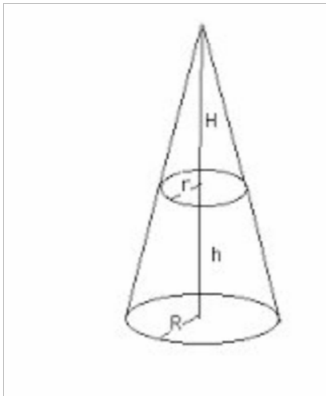


### Example

Find the surface area of a fabric required to make a lampshade in the form of a frustum whose top and bottom diameters are 20 cm and 30 cm respectively and height 12 cm.

### Solution

Complete the cone from which the frustum is made, by adding a smaller cone of height  $x$  cm.



$h = 12$ ,  $H = x$  cm,  $r = 10$  cm,  $R = 15$  cm

From the knowledge of similar  $\frac{x}{10} = \frac{x+12}{15}$

$$15x = 10x + 120$$

$$15x - 10x = 120$$

$$5x = 120$$

$$x = 24$$

Surface area of a frustum = area of the curved surface of bigger cone - area of curved surface of smaller cone.

$$L = 24 + 12 = 36 \text{ cm}$$

$$\text{Surface area} = \pi RL - \pi rl$$

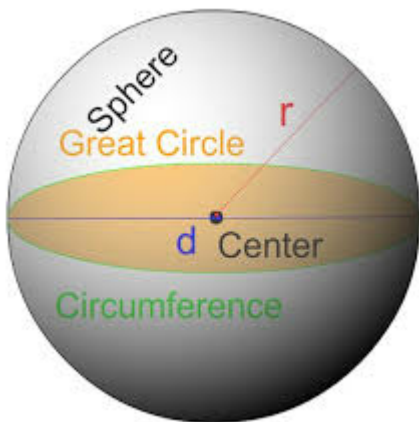
$$= \left( \frac{22}{7} \times 15 \times \sqrt{36^2 + 15^2} \right) - \left( \frac{22}{7} \times 10 \times \sqrt{24^2 + 10^2} \right)$$

$$= 1838.57 \text{ cm}^2 - 817.14 \text{ cm}^2$$

$$= 1021 \text{ cm}^2 \text{ 4 s.f}$$

## Surface area of the sphere

A sphere is solid that it's entirely round with every point on the surface at equal distance from the Centre. Surface area is =  $4\pi r^2$



## Example

Find the surface area of a sphere whose diameter is equal to 21 cm

## Solution

$$\text{Surface area} = 4\pi r^2$$

$$= 4 \times 3.14 \times 10.5 \times 10.5$$

$$= 1386 \text{ cm}^2$$

End of topic

Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

## Past KCSE Questions on the topic.

1. A swimming pool water surface measures 10m long and 8m wide. A path of uniform width is made all round the swimming pool. The total area of the water surface and the path is  $168\text{m}^2$

(a) Find the width of the path (4 mks)

(b) The path is to be covered with square concrete slabs. Each corner of the path is covered with a slab whose side is equal to the width of the path. The rest of the path is covered with slabs of side 50cm. The cost of making each corner slab is sh 600 while the cost of making each smaller slab is sh.50. Calculate

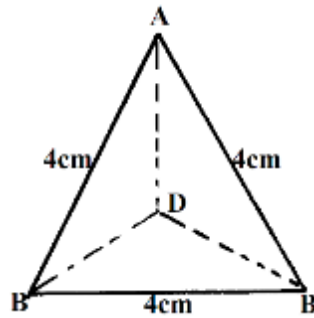
(i) The number of the smaller slabs used (4 mks)

(ii) The total cost of the slabs used to cover the whole path (2 mks)

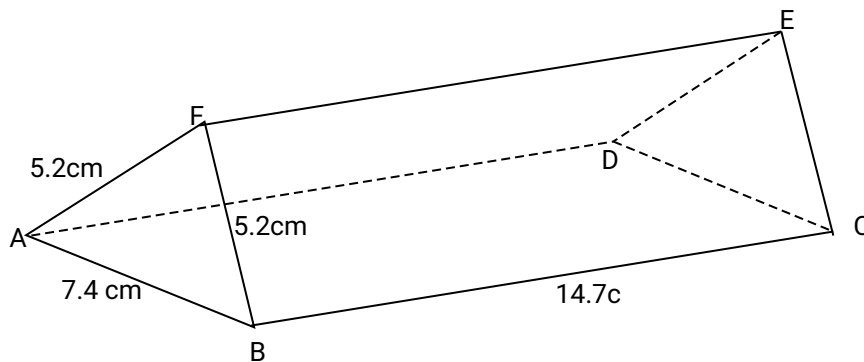
2. The figure below shows a solid regular tetrapack of sides 4cm.

(a) Draw a labelled net of the solid. (1mk)

(b) Find the surface area of the solid. (2mks)

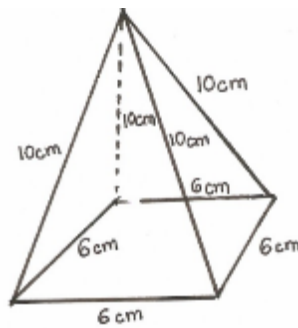


3. The diagram shows a right glass prism ABCDEF with dimensions as shown.

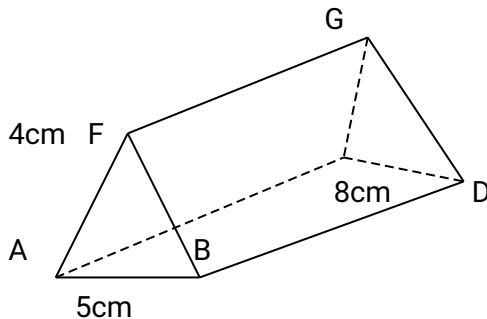


**Calculate:**

- (a) the perimeter of the prism (2 mks)
  - (b) The total surface area of the prism (3 mks)
  - (c) The volume of the prism (2 mks)
  - (d) The angle between the planes AFED and BCEF (3 mks)
4. The base of a rectangular tank is 3.2m by 2.8m. Its height is 2.4m. It contains water to a depth of 1.8m. Calculate the surface area inside the tank that is not in contact with water. (2mks)
5. Draw the net of the solid below and calculate surface area of its faces (3mks)

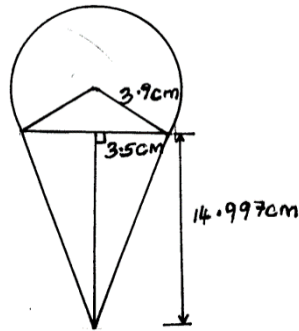


6.



The figure above is a triangular prism of uniform cross-section in which  $AF = 4\text{cm}$ ,  $AB = 5\text{cm}$  and  $BC = 8\text{cm}$ .

- (a) If angle  $BAF = 30^\circ$ , calculate the surface area of the prism. (3 marks)
  - (b) Draw a clearly labeled net of the prisms. (1 mark)
7. Mrs. Dawati decided to open a confectionary shop at corner Baridi. She decorated its entrance with 10 models of cone ice cream, five on each side of the door. The model has the following shape and dimensions. Using  $\pi = 3.142$  and calculations to 4 d.p.

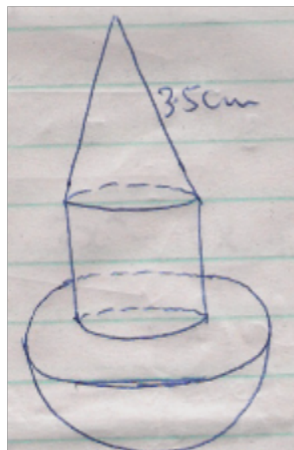


- (a) Calculate the surface area of the conical part. (2mks)
- (b) Calculate the surface area of the top surface. (4mks)
- (c) Find total surface area of one model. (2mks)
- (d) If painting  $5\text{cm}^2$  cost ksh 12.65, find the total cost of painting the models (answer to 1 s.f). (2mks)

8. A right pyramid of height 10cm stands on a square base ABCD of side 6 cm.

- a) Draw the net of the pyramid in the space provided below. (2mks)
- b) Calculate:-
  - (i) The perpendicular distance from the vertex to the side AB. (2mks)
  - (ii) The total surface area of the pyramid. (4mks)
- c) Calculated the volume of the pyramid. (2mks)

9. The figure below shows a solid object consisting of three parts. A conical part of radius 2 cm and slant height 3.5 cm a cylindrical part of height 4 cm. A hemispherical part of radius 3 cm . the cylinder lies at the centre of the hemisphere. ( $\pi = 3.142$ )



Calculate to four significant figures:

- I. The surface area of the solid (5 marks)
- II. The volume of the solid (5 marks)

10. A lampshade is in the form of a frustum of a cone. Its bottom and top diameters are 12cm and 8cm respectively. Its height is 6cm. Find;

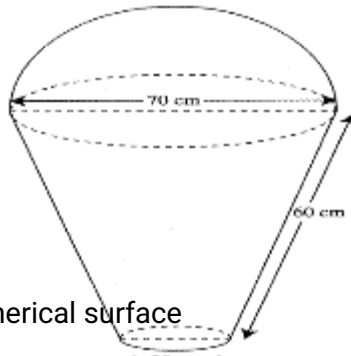
(a) The area of the curved surface of the lampshade

(b) The material used for making the lampshade is sold at Kshs.800 per square metres. Find the cost of ten lampshades if a lampshade is sold at twice the cost of the material

11. A cylindrical piece of wood of radius 4.2cm and length 150cm is cut lengthwise into two equal pieces. Calculate the surface area of one piece

12. The base of an open rectangular tank is 3.2m by 2.8m. Its height is 2.4m. It contains water to a depth of 1.8m. Calculate the surface area inside the tank that is not in contact with water

13. The figure below represents a model of a solid structure in the shape of frustum of a cone with a hemispherical top. The diameter of the hemispherical part is 70cm and is equal to the diameter of the top of the frustum. The frustum has a base diameter of 28cm and slant height of 60cm.



**Calculate:**

- (a) The area of the hemispherical surface
- (b) The slant height of cone from which the frustum was cut
- (c) The surface area of frustum
- (d) The area of the base
- (e) The total surface area of the model

14. A room is 6.8m long, 4.2m wide and 3.5m high. The room has two glass doors each measuring 75cm by 2.5m and a glass window measuring 400cm by 1.25m. The walls are to be painted except the window and doors.

a) Find the total area of the four walls

b) Find the area of the walls to be painted

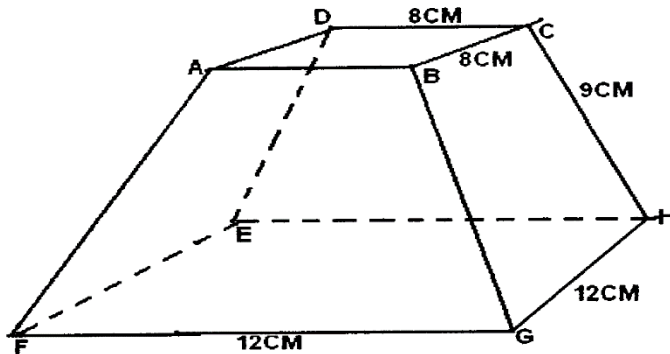
c) Paint **A** costs Shs.80 per litre and paint **B** costs Shs.35 per litre. 0.8 litres of **A** covers an area of  $1\text{m}^2$  while  $0.5\text{m}^2$  uses 1 litre of paint **B**. If two coats of each paint are to be applied. Find the cost of painting the walls using:

i) Paint **A**

ii) Paint **B**

d) If paint A is packed in 400ml tins and paint B in 1.25litres tins, find the least number of tins of each type of paint that must be bought.

15. The figure below shows a solid frustrum of pyramid with a square top of side 8cm and a square base of side 12cm. The slant edge of the frustrum is 9cm



Calculate:

- (a) The total surface area of the frustrum
- (b) The volume of the solid frustrum
- (c) The angle between the planes BCHG and the base EFGH.

## CHAPTER THIRTY SEVEN

### VOLUME OF SOLIDS

#### Specific Objectives

By the end of the topic the learner should be able to:

- a.) Find the volume of a prism
- b.) Find the volume of a pyramid
- c.) Find the volume of a cone
- d.) Find the volume of a frustum
- e.) Find the volume of a sphere and a hemisphere.

#### Content

Volumes of prisms, pyramids, cones, frustums and spheres.

## Introduction

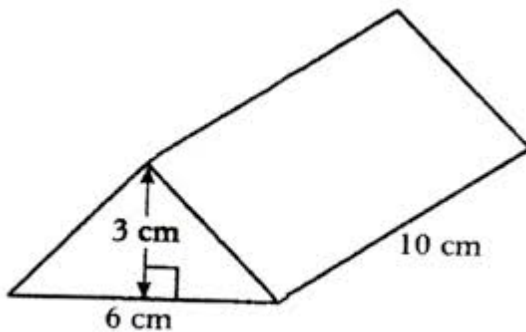
Volume is the amount of space occupied by an object. It's measured in cubic units.

Generally volume of objects is base area x height

## Volume of a Prism

A prism is a solid with uniform cross section .The volume  $V$  of a prism with cross section area  $A$  and length  $l$  is given by  $V = AL$

## Example



## Solution

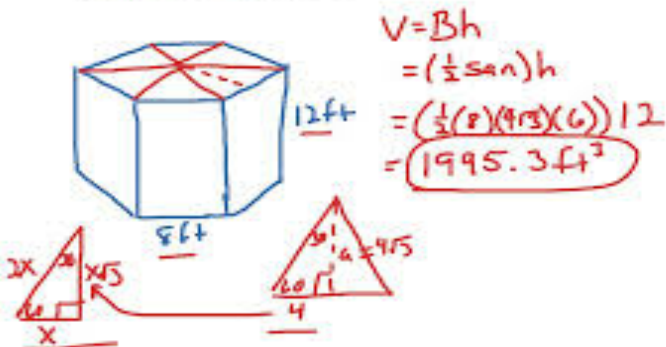
Volume of the prism = base area x length (base is triangle)

$$= \frac{1}{2} \times 6 \times 3 \times 10$$

$$= 90\text{cm}^2$$

## Example

Find the volume of a regular hexagonal prism with edge length 8 ft. and height 12 ft.



## Explanation

A cross- sectional area of the hexagonal is made up of 6 equilateral triangles whose sides are 8 ft

To find the height we take one triangle as shown above

Using sine rule we get the height

### Solution

$$\text{Area of cross section} = 6 \times \frac{1}{2} \times 8 \times 8 \sin 60$$

$$= 166.28$$

$$\text{Volume} = 166.28 \times 12$$

$$= 1995.3 \text{ ft}^2$$

### Volume of a pyramid

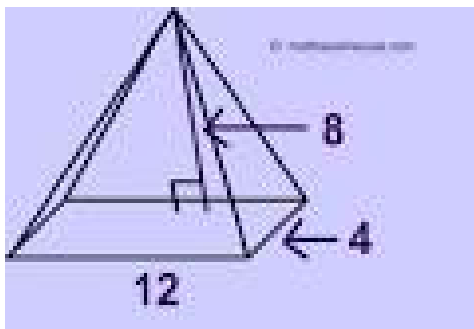
$$\text{Volume of a pyramid} = \frac{1}{3} Ah$$

Where A = area of the base and h = vertical height

### Example

Find the volume of a pyramid with the vertical height of 8 cm and width 4 cm length 12 cm.

### Solution.



$$\text{Volume} = \frac{1}{3} \times 12 \times 4 \times 8$$

$$= 128 \text{ cm}^2$$

### Volume of a sphere

$$V = \frac{4\pi r^3}{3}$$

## Volume of a cone

$$\text{Volume} = \frac{1}{3} \text{ area of base } \times \text{ height}$$

$$= \frac{1}{3} \pi r^2 h$$

## Example

Calculate the volume of a cone whose height is 12 cm and length of the slant height is 13 cm

## Solution

$$\text{Volume} = \frac{1}{3} (\text{base area} \times \text{height})$$

$$= \frac{1}{3} \pi r^2 h$$

$$\text{But, base radius } r = \sqrt{13^2 - 12^2} = \sqrt{25} = 5$$

$$\text{Therefore volume} = \frac{1}{3} \times \frac{22}{7} \times 25 \times 12 \text{ cm}$$

$$= 314.3 \text{ cm}^2$$

## Volume of a frustrum

Volume = volume of large cone – volume of smaller cone

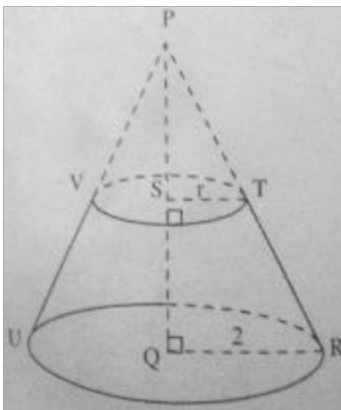
## Example

A frustrum of base radius 2 cm and height 3.6 cm. if the height of the cone from which it was cut was 6 cm, calculate

The radius of the top surface

The volume of the frustrum

## Solution



Triangles PST and PQR are similar

$$\text{Therefore } \frac{PQ}{PS} = \frac{QR}{ST} = \frac{PR}{PT}$$

$$\text{Hence } \frac{6}{2.4} = \frac{2}{ST}$$

$$ST = 0.8 \text{ cm}$$

The radius of the top surface is 0.8 cm

Volume of the frustum = volume of large cone – volume of smaller cone

$$\begin{aligned} &= \frac{1}{3} \times 3.142 \times 4 \times 6 - \frac{1}{3} \times 3.142 \times (0.8^2) \times 2.4 \\ &= 25.14 - 1.61 = 23.53 \text{ cm}^2 \end{aligned}$$

End of topic

Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

## Past KCSE Questions on the topic.

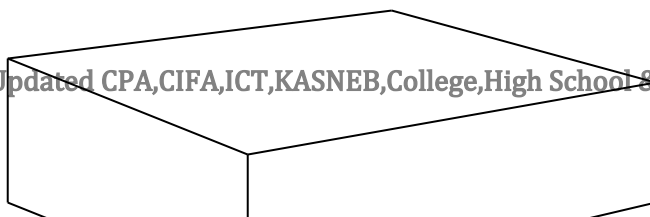
1. Metal cube of side 4.4cm was melted and the molten material used to make a sphere. Find to 3

significant figures the radius of the sphere  $\left( \text{take } \pi = \frac{22}{7} \right)$  (3mks)

2. Two metal spheres of diameter 2.3cm and 3.86cm are melted. The molten material is used to cast equal cylindrical slabs of radius 8mm and length 70mm.

If  $\frac{1}{20}$  of the metal is lost during casting. Calculate the number of complete slabs casted. (4mks)

3. The volume of a rectangular tank is  $256\text{cm}^3$ . The dimensions are as in the figure.



$\frac{1}{4}x$

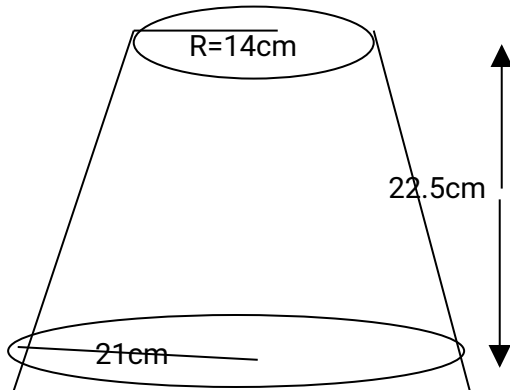
$x-8$

16cm

Find the value of x

(3 marks)

4.



The diagram represent a solid frustum with base radius  $21\text{cm}$  and top radius  $14\text{cm}$ . The frustum is  $22.5\text{cm}$  high and is made of a metal whose density is  $3\text{g/cm}^3$   $\pi = \frac{22}{7}$ .

a) Calculate

(i) the volume of the metal in the frustum.

(5 marks)

(ii) the mass of the frustum in kg.

(2 marks)

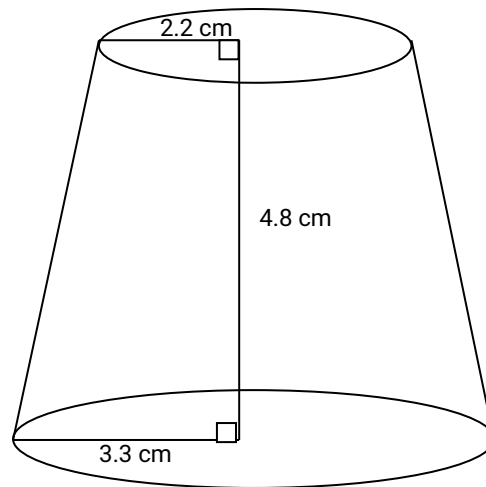
b) The frustum is melted down and recast into a solid cube. In the process 20% of the metal is lost.

Calculate to 2 decimal places the length of each side of the cube.

(3 marks)

5.

The figure below shows a frustum

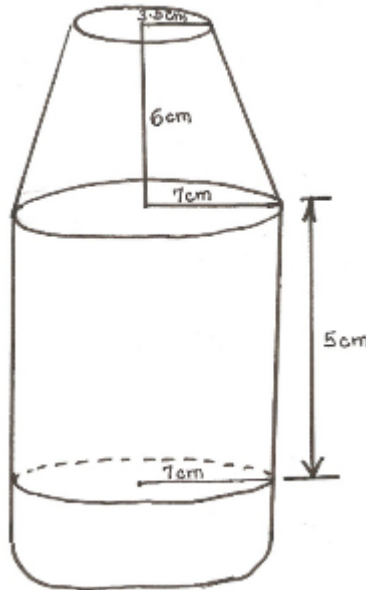


Find the volume of the frustrum

(4 mks)

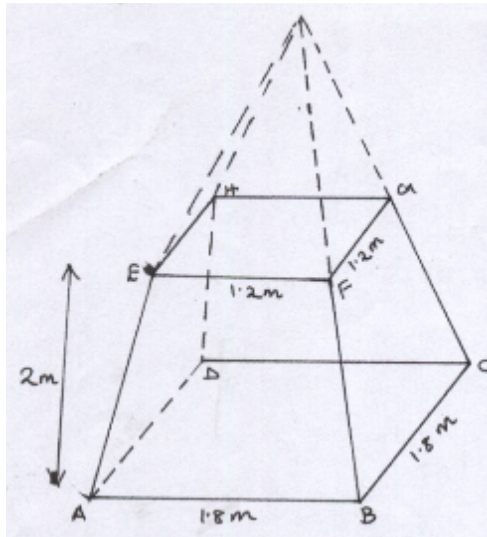
6. The formula for finding the volume of a sphere is given by  $V = \frac{4}{3} \pi r^3$ . Given that  $V = 311$  and  $\pi = 3.142$ , find  $r$ . (3 mks)

7. A right conical frustrum of base radius 7cm and top radius 3.5cm, and height of 6cm is stuck onto a cylinder of base radius 7cm and height 5cm which is further attached to a hemisphere to form a closed solid as shown below



Find:

- (a) The volume of the solid (5mks)
- (b) The surface area of the solid (5mks)
8. A lampshade is made by cutting off the top part of a square-based pyramid VABCD as shown in the figure below. The base and the top of the lampshade have sides of length 1.8m and 1.2m respectively. The height of the lampshade is 2m



Calculate

- a) The volume of the lampshade (4mks)
- b) The total surface area of the slant surfaces (4mks)
- c) The angle at which the face BCGF makes with the base ABCD. (2mks)

9. A solid right pyramid has a rectangular base 10cm by 8cm and slanting edge 16cm.

calculate:

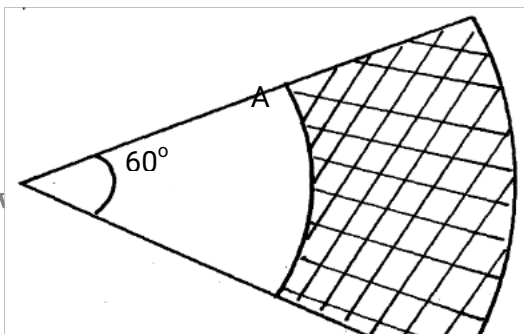
- (a) The vertical height
- (b) The total surface area
- (c) The volume of the pyramid

10. A solid cylinder of radius 6cm and height 12cm is melted and cast into spherical balls of radius 3cm. Find the number of balls made

11. The sides of a rectangular water tank are in the ratio 1: 2:3. If the volume of the tank is  $1024\text{cm}^3$ . Find the dimensions of the tank. (4s.f)

12. The figure below represents sector OAC and OBD with radius OA and OB respectively.

Given that OB is twice OA and angle AOC =  $60^\circ$ . Calculate the area of the shaded region in  $\text{m}^2$ , given that OA = 12cm

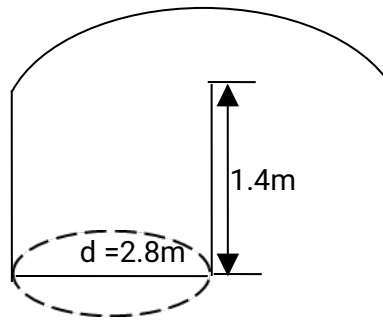


O

C

D

13. The figure below shows a closed water tank comprising of a hemispherical part surmounted on top of a cylindrical part. The two parts have the same diameter of 2.8m and the cylindrical part is 1.4m high as shown:-

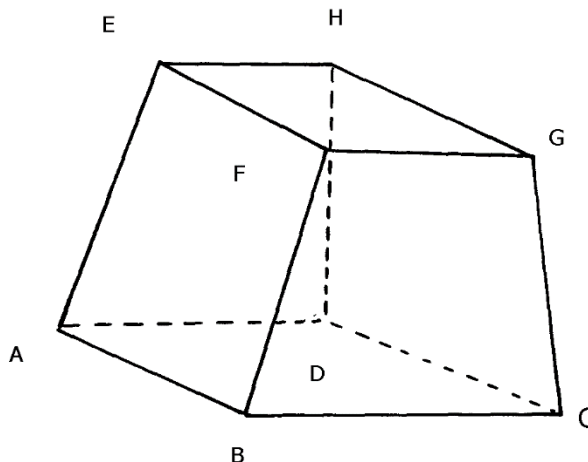


(a) Taking  $\pi = \frac{22}{7}$ , calculate:

- (i) The total surface area of the tank
- (ii) the cost of painting the tank at shs.75 per square metre
- (iii) The capacity of the tank in litres

(b) Starting with the full tank, a family uses water from this tank at the rate of 185litres/day for the first 2days. After that the family uses water at the rate of 200 liters per day. Assuming that no more water is added, determine how many days it takes the family to use all the water from the tank since the first day

14. The figure below represents a frustrum of a right pyramid on a square base. The vertical height of the frustrum is 3 cm. Given that  $EF = FG = 6$  cm and that  $AB = BC = 9$  cm

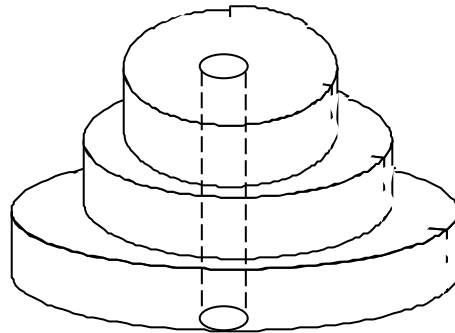


Calculate;

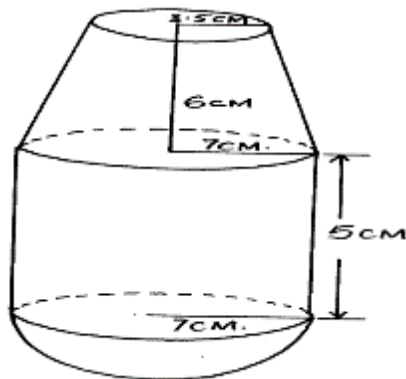
- a) The vertical height of the pyramid.
- b) The surface area of the frustrum.
- c) Volume of the frustrum.
- d) The angle which line AE makes with the base ABCD.

15. A metal hemisphere of radius 12cm is melted done and recast into the shape of a cone of base radius 6cm. Find the perpendicular height of the cone

16. A solid consists of three discs each of  $1\frac{1}{2}$  cm thick with diameter of 4 cm, 6 cm and 8 cm respectively. A central hole 2 cm in diameter is drilled out as shown below. If the density of material used is  $2.8 \text{ g/cm}^3$ , calculate its mass to 1 decimal place



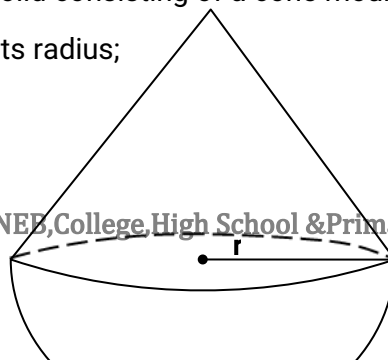
17. A right conical frustrum of base radius 7 cm and top radius 3.5 cm and height 6 cm is stuck onto a cylinder of base radius 7 cm and height 5 cm which is further attached to form a closed solid as shown below.



Find;

- a) The volume of the solid.
- b) The surface area of the solid.

18. The diagram below shows a metal solid consisting of a cone mounted on hemisphere. The height of the cone is  $1\frac{1}{2}$  times its radius;



Given that the volume of the solid is  $31.5\pi \text{ cm}^3$ , find:

(a) The radius of the cone

(b) The surface area of the solid

(c) How much water will rise if the solid is immersed totally in a cylindrical container which contains some water, given the radius of the cylinder is 4cm

(d) The density, in  $\text{kg/m}^3$  of the solid given that the mass of the solid is 144gm

19. A solid metal sphere of volume  $1280 \text{ cm}^3$  is melted down and recast into 20 equal solid cubes. Find the length of the side of each cube. Calculate the volume of the frustum

## Specific Objectives

By the end of the topic the learner should be able to:

- a.) Expand algebraic expressions that form quadratic equations
- b.) Derive the three quadratic identities
- c.) Identify and use the three quadratic identities
- d.) Factorize quadratic expressions including the identities
- e.) Solve quadratic equations by factorization
- f.) Form and solve quadratic equations.

## Content

- a.) Expansion of algebraic expressions to form quadratic expressions of the form  $ax^2 + bx + c$ , where  $a$ ,  $b$  and  $c$  are constants
- b.) The three quadratic identities:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

- c.) Using the three quadratic identities
- d.) Factorisation of quadratic expressions
- e.) Solve quadratic equations by factorization
- f.) Form and solve quadratic equations.

## Introduction

### Expansion

A quadratic is any expression of the form  $ax^2 + bx + c$ ,  $a \neq 0$ . When the expression  $(x + 5)(3x + 2)$  is

written in the form,  $3x^2 + 17x + 10$ , it is said to have been expanded

### Example

Expand  $(m + 2n)(m - n)$

### Solution

Let  $(m - n)$  be a

$$\begin{aligned} \text{Then } (m + 2n)(m - n) &= (m + 2n)a \\ &= ma + 2na \\ &= m(m - n) + 2n(m - n) \\ &= m^2 - mn + 2mn - 2n^2 \\ &= m^2 + mn - 2n^2 \end{aligned}$$

### Example

Expand  $(\frac{1}{4} - \frac{1}{x})^2$

### Solution

$$\begin{aligned} (\frac{1}{4} - \frac{1}{x})^2 &= (\frac{1}{4} - \frac{1}{x})(\frac{1}{4} - \frac{1}{x}) \\ &= \frac{1}{4}(\frac{1}{4} - \frac{1}{x}) - \frac{1}{x}(\frac{1}{4} - \frac{1}{x}) \\ &= \frac{1}{16} - \frac{1}{4x} - \frac{1}{4x} + \frac{1}{x^2} \\ &= \frac{1}{16} - \frac{1}{2x} + \frac{1}{x^2} \end{aligned}$$

### The quadratic identities.

$$(a + b)^2 = (a^2 + 2ab + b^2)$$

$$(a - b)^2 = (a^2 - 2ab + b^2)$$

$$(a + b)(a - b) = (a^2 - b^2)$$

### Examples

$$(X+2)^2 \quad x^2+4x+4$$

$$(X-3)^2 \quad x^2-6x+9$$

$$(X+ 2a)(X -2a) \quad x^2- 4x^2$$

## Factorization

To factorize the expression ,  $ax^2 + bx + c$  ,we look for two numbers such that their product is  $ac$  and their sum is  $b$ .  $a$  ,  $b$  are the coefficient of  $x$  while  $c$  is the constant

### Example

$$8x^2 + 10x + 3$$

### Solution

Look for two number such that their product is  $8 \times 3 = 24$ .

Their sum is 10 where 10 is the coefficient of  $x$ ,

The number are 4 and 6,

Rewrite the term  $10x$  as  $4x + 6x$ , thus  $8x^2 + 4x + 6x + 3$

Use the grouping method to factorize the expression

$$= 4x(2x + 1) + 3(2x + 1)$$

$$= (4x + 3)(2x + 1)$$

### Example

Factorize

$$6x^2 - 13x + 6$$

### Solution

Look for two number such that the product is  $6 \times 6 = 36$  and the sum is  $-13$ .

The numbers are  $-4$  and  $-9$

Therefore,  $6x^2 - 13x + 6$

$$= 6x^2 - 4x - 9x + 6$$

$$= 2x(3x - 2) - 3(3x - 2)$$

$$= (2x - 3)(3x - 2)$$

## Quadratic Equations

In this section we are looking at solving quadratic equation using factor method.

### Example

Solve  $x^2 + 3x - 54 = 0$

### Solution

Factorize the left hand side

$$x^2 + 3x - 54 = x^2 - 6x + 9x - 54 = 0$$

### Note;

The product of two numbers should be  $-54$  and the sum  $3$

$$\begin{aligned}
 &=x^2 - 6x + 9x - 54 \\
 &= x(x-6)(x+9) = 0 \\
 &= (x-6)(x+9) = 0 \\
 X - 6 = 0, x + 9 = 0
 \end{aligned}$$

Hence  $x = -9$  or  $x = 6$

### Example

Expand the following expression and then factorize it

$$(3x + y)^2 - (x - 3y)^2$$

### Solution

$$(3x + y)^2 - (x - 3y)^2 = 9x^2 + 6xy + y^2 - (x^2 - 6xy + 9y^2)$$

$$= 9x^2 + 6xy + y^2 - x^2 + 6xy + 9y^2$$

$$= 8x^2 + 12xy - 8y^2$$

$= 4(2x^2 + 3xy - 2y^2)$  (You can factorize this expression further, find two numbers whose product is  $4x^2y^2$  and sum is  $3xy$ )

The numbers are  $4xy$  and  $-ay$

$$= 4(2x^2 + 4xy - xy - 2y^2)$$

$$= 4 [ 2x(x+2y) - y(x+2y) ]$$

$$= 4 (x + 2y)(2x - y)$$

## Formation of Quadratic Equations

### Given the roots

Given that the roots of quadratic equations are  $x = 2$  and  $x = -3$ , find the quadratic equation

If  $x = 2$ , then  $x - 2 = 0$

If  $x = -3$ , then  $x + 3 = 0$

Therefore,  $(x - 2)(x + 3) = 0$

$$x^2 + x - 6 = 0$$

### Example

A rectangular room is 4 m longer than it is wide. If its area is  $12 \text{ m}^2$  find its dimensions.

### Solution

Let the width be  $x \text{ m}$ . its length is then  $(x + 4) \text{ m}$ .

The area of the room is  $x(x+4)\text{m}^2$

Therefore  $x(x + 4) = 12$

$$x^2 + 4x = 12$$

$$x^2 + 4x - 12 = 0$$

$$(x + 6)(x - 2) = 0$$

$$(x+6) = 0 \quad (x-2) = 0 \quad \text{therefore } x = -6 \text{ or } 2$$

-6 is being ignored because length cannot be negative

The length of the room is  $x + 4 = 2 + 4$

$$= 6 \text{ m}$$

End of topic

Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

## Past KCSE Questions on the topic.

1. Simplify  $\frac{2y^2 - xy - x^2}{2x^2 - 2y^2}$  (3mks)

2. Solve the following quadratic equation giving your answer to 3 d.p. (3mks)

$$\frac{23}{x} - \frac{1}{x^2} - 120 = 0.$$

3. Simplify (3 mks)

$$\frac{16x^2 - 4}{4x^2 + 2x - 2} \div \frac{2x - 2}{x + 1}$$

4. Simplify as simple as possible  $\frac{(4x + 2y)^2 - (2y - 4x)^2}{(2z + y)^2 - (y - 2x)^2}$

5. The sum of two numbers  $x$  and  $y$  is 40. Write down an expression, in terms of  $x$ , for the sum of the squares of the two numbers. Hence determine the minimum value of  $x^2 + y^2$

6. Mary has 21 coins whose total value is Kshs 72. There are twice as many five shillings coins as there are ten shillings coins. The rest one shilling coins. Find the number of ten shilling coins that Mary has.

7. Four farmers took their goats to the market Mohamed had two more goats than Ali Koech had 3 times as many goats as Mohamed. Whereas Odupoy had 10 goats less than both Mohamed and Koech.
- I.) Write a simplified algebraic expression with one variable. Representing the total number of goats
  - II.) Three butchers bought all the goats and shared them equally. If each butcher got 17 goats. How many did Odupoy sell to the butchers?

## CHAPTER THIRTY NINE

# LINEAR INEQUALITIES

### Specific Objectives

By the end of the topic the learner should be able to:

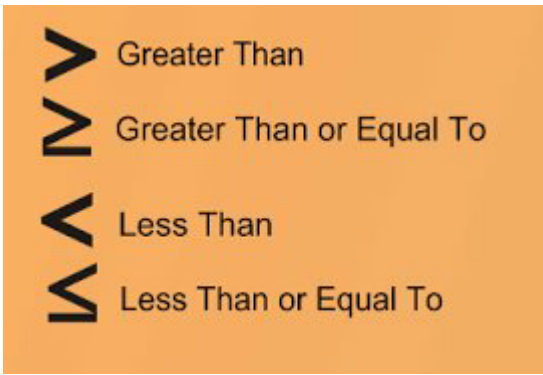
- a.) Identify and use inequality symbols
- b.) Illustrate inequalities on the number line
- c.) Solve linear inequalities in one unknown
- d.) Represent the linear inequalities graphically
- e.) Solve the linear inequalities in two unknowns graphically
- f.) Form simple linear inequalities from inequality graphs.

### Contents

- a.) Inequalities on a number line
- b.) Simple and compound inequality statements e.g.  $x > a$  and  $x < b \Rightarrow a < x < b$
- c.) Linear inequality in one unknown
- d.) Graphical representation of linear inequalities
- e.) Graphical solutions of simultaneous linear inequalities
- f.) Simple linear inequalities from inequality graphs.

## Introduction

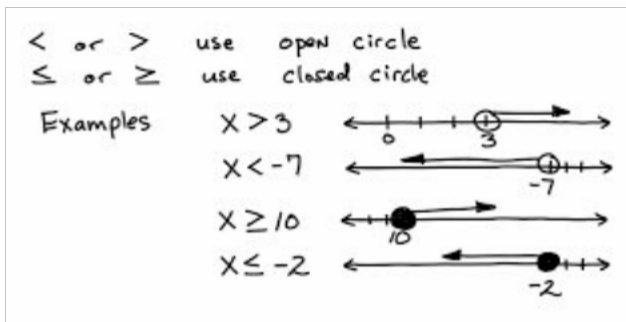
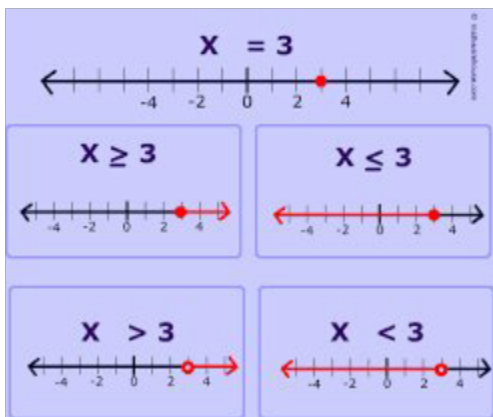
Inequality symbols



Statements connected by these symbols are called **inequalities**

### Simple statements

Simple statements represents only one condition as follows



$X = 3$  represents specific point which is number 3, while  $x > 3$  does not it represents all numbers to the right of 3 meaning all the numbers greater than 3 as illustrated above.  $X < 3$  represents all numbers to left of 3 meaning all the numbers less than 3. The empty circle means that 3 is not included in the list of numbers to greater or less than 3.

The expression  $x \geq 3$  or  $x \leq 3$  means that means that 3 is included in the list and the circle is shaded to show that 3 is included.

### Compound statement

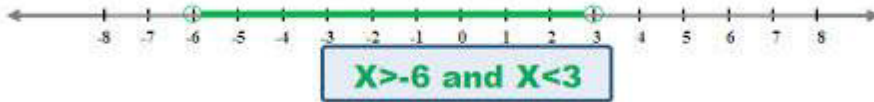
A compound statement is a two simple inequalities joined by "and" or "or." Here are two examples.

$3 \geq x$  and  $x > -3$  Combined into one to form  $-3 < x \leq 3$



All real numbers that are greater than - 3 but less or equal to 3

$$x > -6 \text{ and } x < 3 \text{ forms } -6 < x < 3$$



All real numbers that greater than - 6 but less than 3

## Solution to simple inequalities

### Example

Solve the inequality

$$x - 1 > 2$$

### Solution

Adding 1 to both sides gives ;

$$X - 1 + 1 > 2 + 1$$

Therefore,  $x > 3$

### Note;

In any inequality you may add or subtract the same number from both sides.

### Example

Solve the inequality.

$$X + 3 < 8$$

### Solution

Subtracting three from both sides gives

$$X + 3 - 3 < 8 - 3$$

$$X < 5$$

### Example

Solve the inequality

$$2x + 3 \leq 5$$

Subtracting three from both sides gives

$$2x + 3 - 3 \leq 5 - 3$$

$$2x \leq 2$$

Divide both sides by 2 gives

$$\frac{2x}{2} \leq \frac{2}{2}$$

$$x \leq 1$$

### Example

Solve the inequality  $\frac{1}{3}x - 2 \geq 4$

### Solution

Adding 2 to both sides

$$\frac{1}{3}x - 2 + 2 \geq 4 + 2$$

$$\frac{1}{3}x \geq 6$$

$$\frac{1}{3}x \times 3 \geq 6 \times 3$$

$$x \geq 18$$

### Multiplication and Division by a Negative Number

Multiplying or dividing both sides of an inequality by positive number leaves the inequality sign unchanged

Multiplying or dividing both sides of an inequality by negative number reverses the sense of the inequality sign.

### Example

Solve the inequality  $1 - 3x < 4$

### Solution

$$-3x - 1 < 4 - 1$$

$$-3x < 3$$

$$\frac{-3x}{-3} > \frac{3}{-3}$$

Note that the sign is reversed  $x > -1$

### Simultaneous inequalities

#### Example

Solve the following

$$3x - 1 > -4$$

$$2x + 1 \leq 7$$

#### Solution

Solving the first inequality

$$3x - 1 > -4$$

$$3x > -3$$

$$X > -1$$

Solving the second inequality

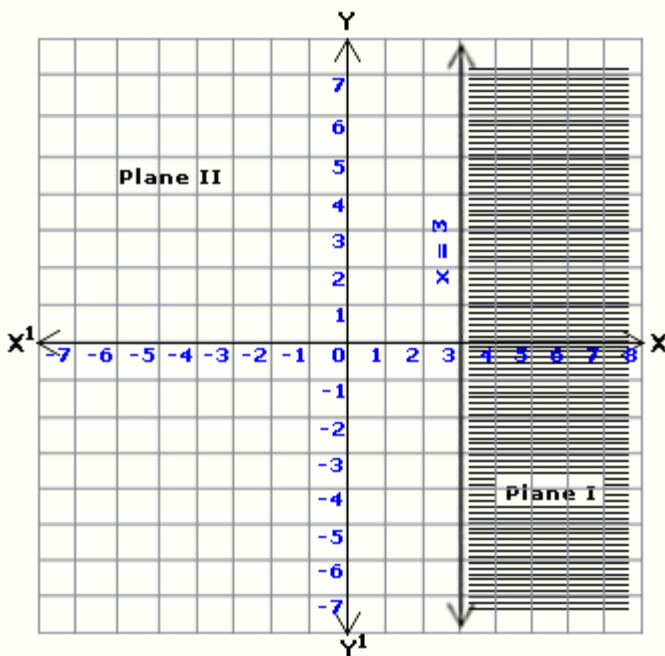
$$2x + 1 \leq 7$$

$2x \leq 6$  Therefore  $x \leq 3$  The combined inequality is  $-1 < x \leq 3$

## Graphical Representation of Inequality

Consider the following;

$$x \leq 3$$



The line  $x = 3$  satisfy the inequality  $\leq 3$  , the points on the left of the line satisfy the inequality.

We don't need the points to the right hence we shade it

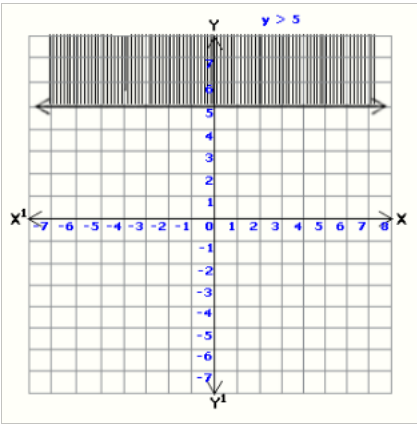
### Note:

We shade the unwanted region

The line is continuous because it forms part of the region e.g it starts at 3.for  $\leq$  or  $\geq$  inequalities the line must be continuous

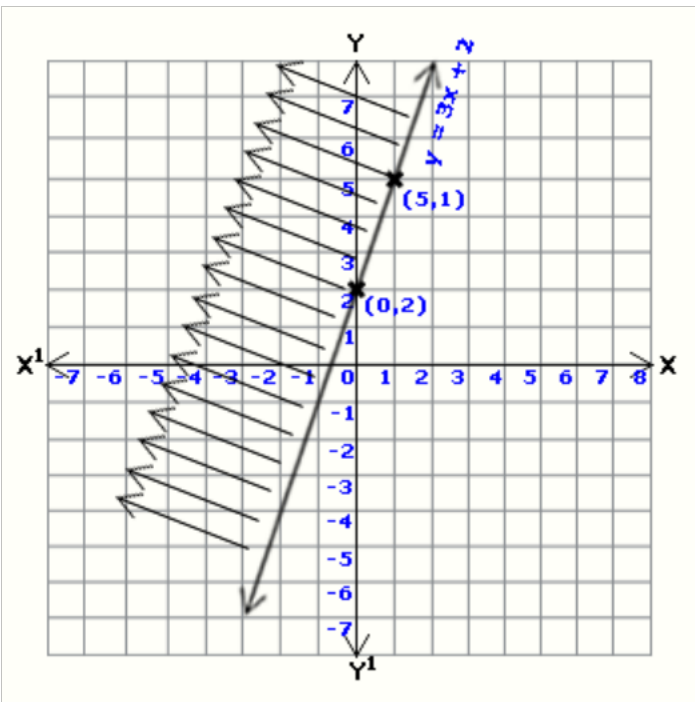
For  $<$  or  $>$  the line is not continuous its dotted.This is because the value on the line does

Not satisfy the inequality.



### Linear Inequality of Two Unknown

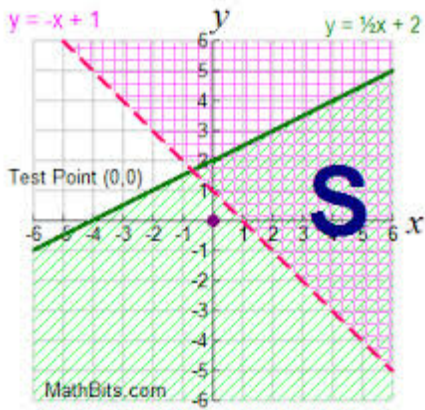
Consider the inequality  $y \leq 3x + 2$  the boundary line is  $y = 3x + 2$



If we pick any point above the line eg  $(-3, 3)$  then substitute in the equation  $y - 3x \leq 2$  we get  $12 \leq 2$  which is not true so the values lies in the unwanted region hence we shade that region .

### Intersecting Regions

These are identities regions which satisfy more than one inequality simultaneously. Draw a region which satisfy the following inequalities  $y + x \geq 1$  and  $y - \frac{1}{2}x \geq 2$



End of topic

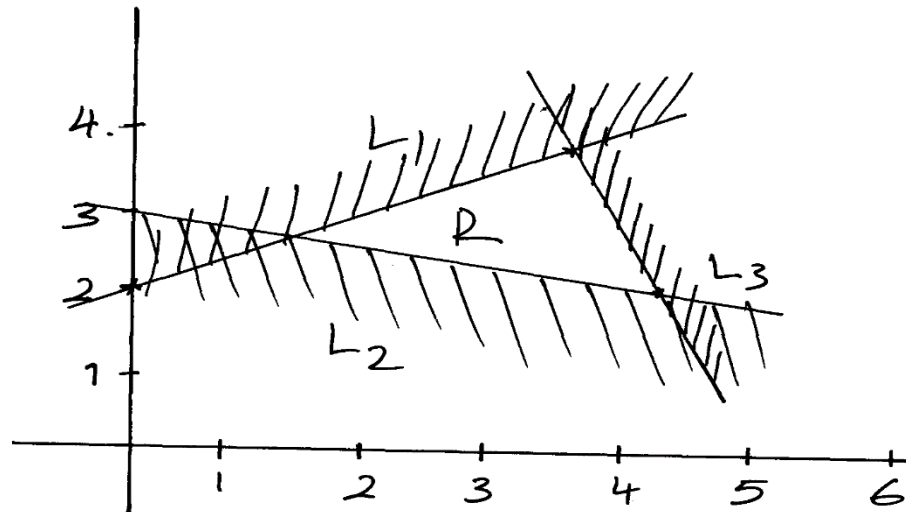
Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

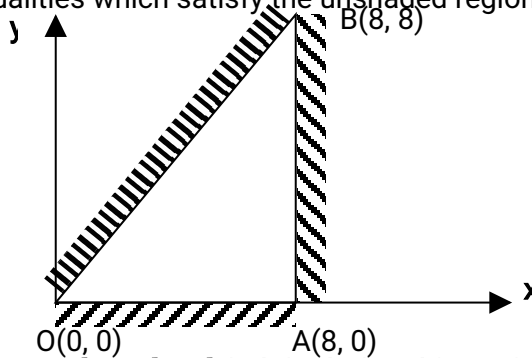
## Past KCSE Questions on the topic.

- Find the range of  $x$  if  $2 \leq 3 - x < 5$
- Find all the integral values of  $x$  which satisfy the inequalities:  
 $2(2-x) < 4x - 9 < x + 11$
- Solve the inequality and show the solution  
 $3 - 2x < x \leq \frac{2x + 5}{3}$  on the number line
- Solve the inequality  $\frac{x - 3}{4} + \frac{x - 5}{6} \leq \frac{4x + 6}{8} - 1$
- Solve and write down all the integral values satisfying the inequality.  
 $X - 9 \leq -4 < 3x - 4$
- Show on a number line the range of all integral values of  $x$  which satisfy the following pair of inequalities:  
 $3 - x \leq 1 - \frac{1}{2}x$   
 $-\frac{1}{2}(x-5) \leq 7-x$

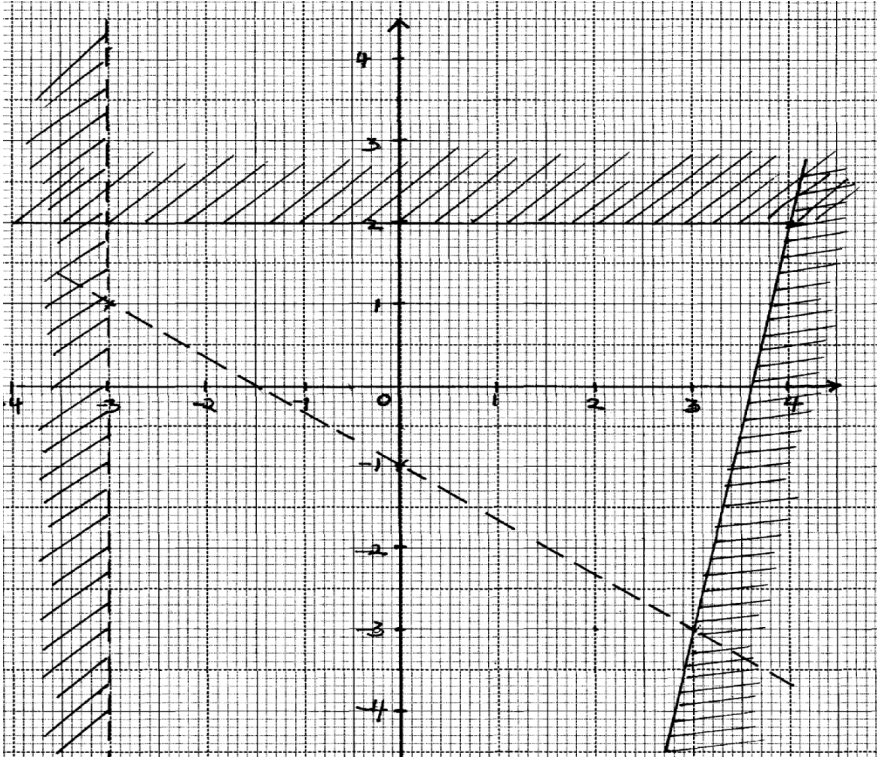
7. Solve the inequalities  $4x - 3 \leq 6x - 1 < 3x + 8$ ; hence represent your solution on a number line
8. Find all the integral values of  $x$  which satisfy the inequalities  
 $2(2-x) < 4x - 9 < x + 11$
9. Given that  $x + y = 8$  and  $x^2 + y^2 = 34$   
 Find the value of:-  
 a)  $x^2 + 2xy + y^2$   
 b)  $2xy$
10. Find the inequalities satisfied by the region labelled R



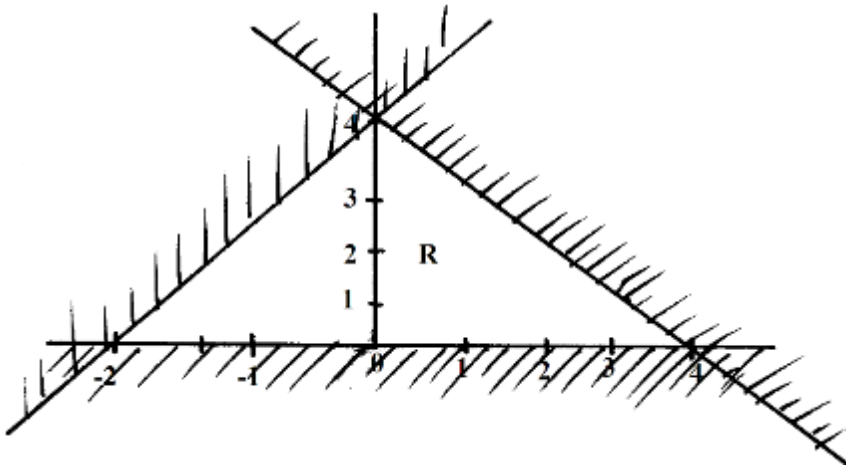
11. The region R is defined by  $x \geq 0, y \geq -2, 2y + x \leq 2$ . By drawing suitable straight line on a sketch, show and label the region R
12. Find all the integral values of  $x$  which satisfy the inequality  
 $3(1+x) < 5x - 11 < x + 45$
13. The vertices of the unshaded region in the figure below are  $O(0, 0)$ ,  $B(8, 8)$  and  $A(8, 0)$ . Write down the inequalities which satisfy the unshaded region



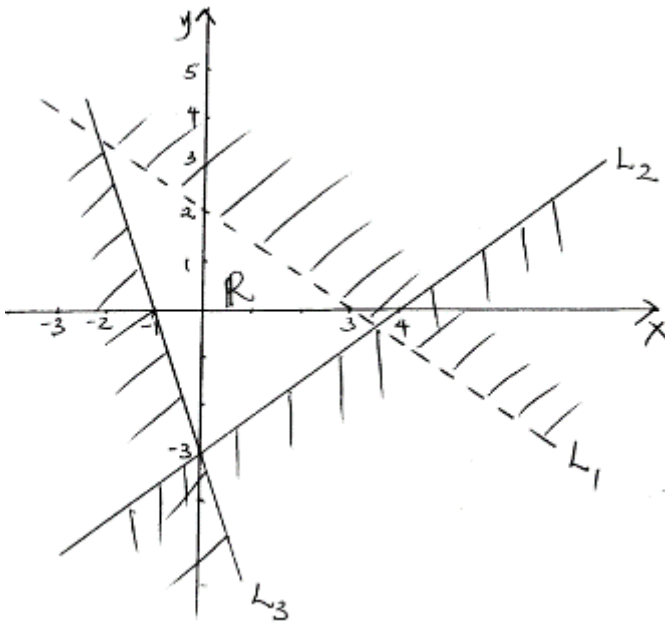
14. Write down the inequalities that satisfy the given region simultaneously. (3mks)



15. Write down the inequalities that define the unshaded region marked R in the figure below. (3mks)



16. Write down all the inequalities represented by the regions R. (3mks)



17. a) On the grid provided draw the graph of  $y = 4 + 3x - x^2$  for the integral values of  $x$  in the interval  $-2 \leq X \leq 5$ . Use a scale of 2cm to represent 1 unit on the  $x$  – axis and 1 cm to represent 1 unit on the  $y$  – axis. (6mks)
- b) State the turning point of the graph. (1mk)
- c) Use your graph to solve.
- (i)  $-x^2 + 3x + 4 = 0$
- (ii)  $4x = x^2$

## CHAPTER FOURTY

### LINEAR MOTION

#### Specific Objectives

By the end of the topic the learner should be able to:

- a.) Define displacement, speed, velocity and acceleration
- a.) Distinguish between:
  - ✓ distance and displacement
  - ✓ speed and velocity
- b.) Determine velocity and acceleration
- c.) Plot and draw graphs of linear motion (distance and velocity time graphs)
- d.) Interpret graphs of linear motion

- e.) Define relative speed
- f.) Solve the problems involving relative speed.

### Content

- a.) Displacement, velocity, speed and acceleration
- b.) Determining velocity and acceleration
- c.) Relative speed
- d.) Distance - time graph
- e.) Velocity time graph
- f.) Interpretation of graphs of linear motion
- g.) Solving problems involving relative speed

## Introduction

Distance between the two points is the length of the path joining them while displacement is the distance in a specified direction

## Speed

$$\text{Average speed} = \frac{\text{distance covered}}{\text{time taken}}$$

## Example

A man walks for 40 minutes at 60 km/hour, then travels for two hours in a minibus at 80 km/hour. Finally, he travels by bus for one hour at 60 km/h. Find his speed for the whole journey .

## Solution

$$\text{Average speed} = \frac{\text{distance covered}}{\text{time taken}}$$

$$\text{Total distance} = \left(\frac{40}{60} \times 60\right) \text{ km} + (2 \times 80) \text{ km} + (1 \times 60) \text{ km} = 260 \text{ km}$$

$$\text{Total time} = \frac{4}{6} + 2 + 1 = 3\frac{2}{3} \text{ hrs}$$

$$\text{Average speed} = \frac{260}{3\frac{2}{3}}$$

$$= \frac{260 \times 3}{11} = 70.9 \text{ km/h}$$

## Velocity and acceleration

For motion under constant acceleration;

$$\text{Average velocity} = \frac{\text{initial velocity} + \text{final velocity}}{2}$$

## Example

A car moving in a given direction under constant acceleration. If its velocity at a certain time is 75 km /h and 10 seconds later its 90 km /hr.

## Solution

$$\begin{aligned} \text{Acceleration} &= \frac{\text{change in velocity}}{\text{time taken}} \\ &= \frac{(90-75) \text{ km/h}}{10 \text{ s}} \\ &= \frac{(90-75) \times 1000}{10 \times 60 \times 60} \text{ m/s}^2 \\ &= \frac{5}{12} \text{ m/s}^2 \end{aligned}$$

## Example

A car moving with a velocity of 50 km/h then the brakes are applied so that it stops after 20 seconds .in this case the final velocity is 0 km/h and initial velocity is 50 km/h.

## Solution

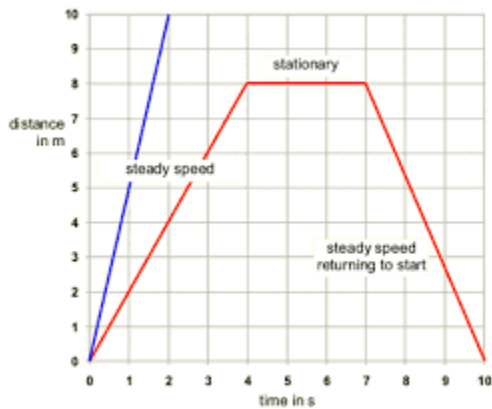
$$\text{Acceleration} = \frac{(0-50) \times 1000}{20 \times 60 \times 60} \text{ m/s}^2$$

$$= -\frac{25}{36} \text{ m/s}^2$$

Negative acceleration is always referred to as deceleration or retardation

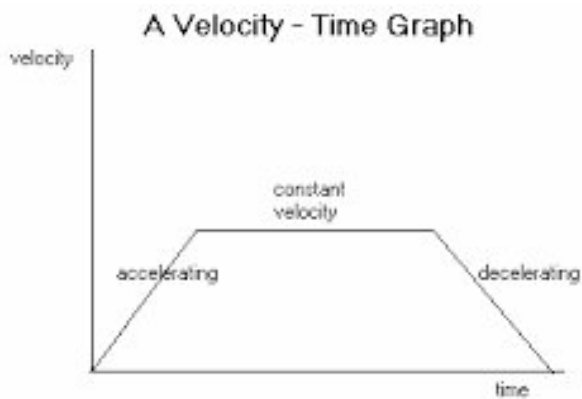
## Distance time graph.

When distance is plotted against time, a distance time graph is obtained.



## Velocity–time Graph

When velocity is plotted against time, a velocity time graph is obtained.



The distance travelled is area under graph.  
The acceleration and deceleration can be found by finding the gradient of the lines.

## Relative Speed

Consider two bodies moving in the same direction at different speeds. Their relative speed is the difference between the individual speeds.

### Example

A van left Nairobi for Kakamega at an average speed of 80 km/h. After half an hour, a car left Nairobi for Kakamega at a speed of 100 km/h.

- a.) Find the relative speed of the two vehicles.
- b.) How far from Nairobi did the car over take the van

### Solution

$$\begin{aligned}\text{Relative speed} &= \text{difference between the speeds} \\ &= 100 - 80 \\ &= 20 \text{ km/h}\end{aligned}$$

Distance covered by the van in 30 minutes

$$\text{Distance} = \frac{30}{60} \times 80 = 40 \text{ km}$$

$$\begin{aligned}\text{Time taken for car to overtake matatu} &= \frac{40}{20} \\ &= 2 \text{ hours}\end{aligned}$$

Distance from Nairobi =  $2 \times 100 = 200 \text{ km}$

### Example

A truck left Nyeri at 7.00 am for Nairobi at an average speed of 60 km/h. At 8.00 am a bus left Nairobi for Nyeri at speed of 120 km/h .How far from nyeri did the vehicles meet if Nyeri is 160 km from Nairobi?

### Solution

$$\begin{aligned}\text{Distance covered by the lorry in 1 hour} &= 1 \times 60 \\ &= 60 \text{ km}\end{aligned}$$

$$\begin{aligned}\text{Distance between the two vehicle at 8.00 am} &= 160 - 60 \\ &= 100 \text{ km}\end{aligned}$$

Relative speed = 60 km/h + 120 km/h

$$\begin{aligned}\text{Time taken for the vehicle to meet} &= \frac{100}{180} \\ &= \frac{5}{9} \text{ hours}\end{aligned}$$

$$\begin{aligned} \text{Distance from Nyeri} &= 60 \times \frac{5}{9} \times 60 \\ &= 60 + 33.3 \\ &= 93.3 \text{ km} \end{aligned}$$

End of topic

Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

## Past KCSE Questions on the topic.

1. A bus takes 195 minutes to travel a distance of  $(2x + 30)$  km at an average speed of  $(x - 20)$  km/h. Calculate the actual distance traveled. Give your answers in kilometers.
- 2.) The table shows the height metres of an object thrown vertically upwards varies with the time  $t$  seconds. The relationship between  $s$  and  $t$  is represented by the equations  $s = at^2 + bt + 10$  where  $b$  are constants.

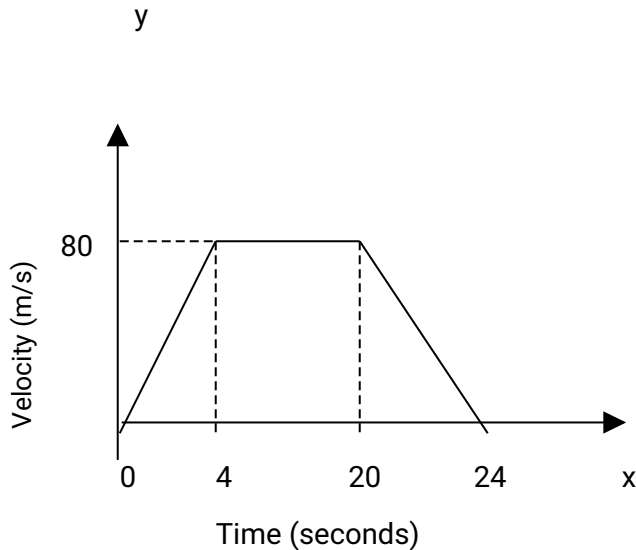
t	0	1	2	3	4	5	6	7	8	9	10
s		45.1									

- I.) Using the information in the table, determine the values of  $a$  and  $b$  ( 2 marks)
  - II.) Complete the table ( 1 mark)
- (b) (i) Draw a graph to represent the relationship between  $s$  and  $t$  ( 3 marks)
  - (ii) Using the graph determine the velocity of the object when  $t = 5$  seconds (2 marks)
- 3.) Two Lorries A and B ferry goods between two towns which are 3120 km apart. Lorry A traveled at km/h faster than lorry B and B takes 4 hours more than lorry A to cover the distance. Calculate the speed of lorry B
  - 4.) A matatus left town A at 7 a.m. and travelled towards a town B at an average speed of 60 km/h. A

second matatus left town B at 8 a.m. and travelled towards town A at 60 km/h. If the distance between the two towns is 400 km, find;

- I.) The time at which the two matatus met
- II.) The distance of the meeting point from town A

5. The figure below is a velocity time graph for a car.



(a) Find the total distance traveled by the car. (2 marks)

(b) Calculate the deceleration of the car. (2 marks)

6. A bus started from rest and accelerated to a speed of 60km/h as it passed a billboard. A car moving in the same direction at a speed of 100km/h passed the billboard 45 minutes later. How far from the billboard did the car catch up with the bus? (3mks)

7. Nairobi and Eldoret are each 250km from Nakuru. At 8.15am a lorry leaves Nakuru for Nairobi. At 9.30am a car leaves Eldoret for Nairobi along the same route at 100km/h. Both vehicles arrive at Nairobi at the same time.

(a) Calculate their time of arrival in Nairobi (2mks)

(b) Find the cars speed relative to that of the lorry. (4mks)

(c) How far apart are the vehicles at 12.45pm. (4mks)

8. Two towns P and Q are 400 km apart. A bus left P for Q. It stopped at Q for one hour and then started the return journey to P. One hour after the departure of the bus from P, a trailer also heading for Q left P. The trailer met the returning bus  $\frac{3}{4}$  of the way from P to Q. They met t hours after the departure of the bus from P.

- (a) Express the average speed of the trailer in terms of  $t$   
(b) Find the ratio of the speed of the bus so that of the trailer.
9. The athletes in an 800 metres race take 104 seconds and 108 seconds respectively to complete the race. Assuming each athlete is running at a constant speed. Calculate the distance between them when the faster athlete is at the finishing line.
10. A and B are towns 360 km apart. An express bus departs from A at 8 am and maintains an average speed of 90 km/h between A and B. Another bus starts from B also at 8 am and moves towards A making four stops at four equally spaced points between B and A. Each stop is of duration 5 minutes and the average speed between any two spots is 60 km/h. Calculate distance between the two buses at 10 am.
11. Two towns A and B are 220 km apart. A bus left town A at 11. 00 am and traveled towards B at 60 km/h. At the same time, a matatu left town B for town A and traveled at 80 km/h. The matatu stopped for a total of 45 minutes on the way before meeting the bus. Calculate the distance covered by the bus before meeting the matatu.
12. A bus travels from Nairobi to Kakamega and back. The average speed from Nairobi to Kakamega is 80 km/hr while that from Kakamega to Nairobi is 50 km/hr, the fuel consumption is 0.35 litres per kilometer and at 80 km/h, the consumption is 0.3 litres per kilometer .Find
- i) Total fuel consumption for the round trip
  - ii) Average fuel consumption per hour for the round trip.
13. The distance between towns M and N is 280 km. A car and a lorry travel from M to N. The average speed of the lorry is 20 km/h less than that of the car. The lorry takes 1h 10 min more than the car to travel from M and N.
- (a) If the speed of the lorry is  $x$  km/h, find  $x$  (5mks)  
(b) The lorry left town M at 8: 15 a.m. The car left town M and overtook the lorry at 12.15 p.m. Calculate the time the car left town M.
14. A bus left Mombasa and traveled towards Nairobi at an average speed of 60 km/hr. after  $2\frac{1}{2}$  hours; a car left Mombasa and traveled along the same road at an average speed of 100 km/ hr. If the distance between Mombasa and Nairobi is 500 km, Determine
- (a) (i) The distance of the bus from Nairobi when the car took off (2mks)  
(ii) The distance the car traveled to catch up with the bus
  - (b) Immediately the car caught up with the bus
  - (c) The car stopped for 25 minutes. Find the new average speed at which the car traveled in order to reach Nairobi at the same time as the bus.
15. A rally car traveled for 2 hours 40 minutes at an average speed of 120 km/h. The car consumes an average of 1 litre of fuel for every 4 kilometers.  
A litre of the fuel costs Kshs 59  
Calculate the amount of money spent on fuel

16. A passenger notices that she had forgotten her bag in a bus 12 minutes after the bus had left. To catch up with the bus she immediately took a taxi which traveled at 95 km/hr. The bus maintained

an average speed of 75 km/ hr. determine

- (a) The distance covered by the bus in 12 minutes
  - (b) The distance covered by the taxi to catch up with the bus
17. The athletes in an 800 metre race take 104 seconds and 108 seconds respectively to complete the race. Assuming each athlete is running at a constant speed. Calculate the distance between them when the faster athlete is at the finishing line.
18. Mwangi and Otieno live 40 km apart. Mwangi starts from his home at 7.30 am and cycles towards Otieno's house at 16 km/ h Otieno starts from his home at 8.00 and cycles at 8 km/h towards Mwangi at what time do they meet?
19. A train moving at an average speed of 72 km/h takes 15 seconds to completely cross a bridge that is 80m long.
- (a) Express 72 km/h in metres per second
  - (b) Find the length of the train in metres

## CHAPTER FOURTY ONE

### STATISTICS (I)

#### Specific Objectives

By the end of the topic the learner should be able to:

- a.) Define statistics
- b.) Collect and organize data
- c.) Draw a frequency distribution table
- d.) Group data into reasonable classes
- e.) Calculate measures of central tendency
- f.) Represent data in form of line graphs, bar graphs, pie-charts, pictogram, histogram and frequency polygons
- g.) Interpret data from real life situations.

## Content

- a.) Definition of statistics
- b.) Collection and organization of data
- c.) Frequency distribution tables (for grouped and ungrouped data)
- d.) Grouping data
- e.) Mean, mode and median for ungrouped and grouped data
- f.) Representation of data: line graph, Bar graph, Pie chart, Pictogram, Histogram, Frequency polygon interpretation of data.

## Introduction

This is the branch of mathematics that deals with the collection, organization, representation and interpretation of data. Data is the basic information.

### Frequency Distribution table

A data table that lists a set of scores and their frequency

#### frequency distribution table

A data table that lists a set of scores and their frequency.

score	tally	frequency ( $f$ )
1		4
2	 	9
3	 	6
4	 	7
5		3
6		2

## Tally

In tallying each stroke represent a quantity.

## Frequency

This is the number of times an item or value occurs.

## Mean

This is usually referred to as arithmetic mean, and is the average value for the data

Data value	Tally	Frequency	Frequency × Data value
2		3	6
3		2	6
4		5	20
5		3	15
6		4	24
7		6	42
8		3	24
9		4	36
	SUM =	30	173

$$\begin{aligned} \text{The mean } (\bar{x}) &= \frac{\text{total marks scored}}{\text{total number of students}} = \frac{\sum fx}{\sum f} \\ &= \frac{173}{30} \\ &= 5.767 \end{aligned}$$

## Mode

This is the most frequent item or value in a distribution or data. In the above table its 7 which is the most frequent.

## Median

To get the median arrange the items in order of size. If there are N items and N is an odd number, the item occupying  $(\frac{n+1}{2})^{\text{th}}$ .

If N is even, the average of the items occupying  $\frac{n}{2}$

## Grouped data

Then difference between the smallest and the biggest values in a set of data is called the range. The data can be grouped into a convenient number of groups called classes. 30 – 40 are called class boundaries.

The class with the highest frequency is called the modal class. In this case its  $50 \leq m < 60$ , the class width or interval is obtained by getting the difference between the class limits. In this case,  $30 - 40 = 10$ , to get the mid-point you divide it by 2 and add it to the lower class limit.

Mass ( $m$ ) kg	Midpoint	Frequency	Midpoint $\times$ Frequency
$30 \leq m < 40$	35	7	245
$40 \leq m < 50$	45	6	270
$50 \leq m < 60$	55	8	440
$60 \leq m < 70$	65	4	260
Totals:		25	1,215

The mean mass in the table above is  $\sum f = 25$ ,  $\sum fx = 1215$

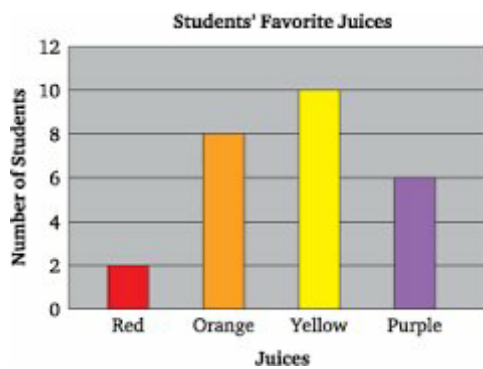
$$\text{Mean } \frac{1215}{25} = 48.6$$

## Representation of statistical data

The main purpose of representation of statistical data is to make collected data more easily understood. Methods of representation of data include.

### Bar graph

Consist of a number of spaced rectangles which generally have major axes vertical. Bars are uniform width. The axes must be labelled and scales indicated.



The students' favorite juices are as follows

Red 2

Orange 8

Yellow 10

Purple 6

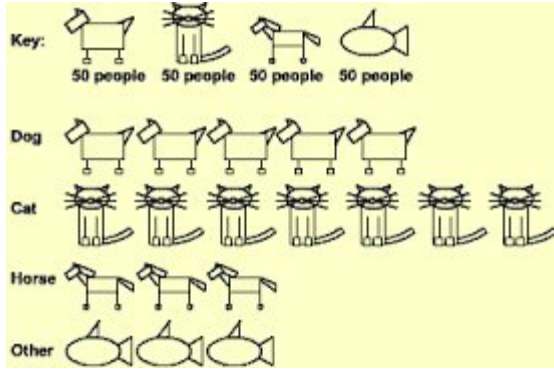
## Pictograms

In a pictogram, data is represented using pictures.

Consider the following data.

The data shows the number of people who love the following animals

Dogs 250, Cats 350, Horses 150 , fish 150



## Pie chart

A pie chart is divided into various sectors .Each sector represent a certain quantity of the item being considered .the size of the sector is proportional to the quantity being measured .consider the export of US to the following countries. Canada \$ 13390, Mexico \$ 8136, Japan \$5824, France \$ 2110 .This information can be represented in a pie chart as follows

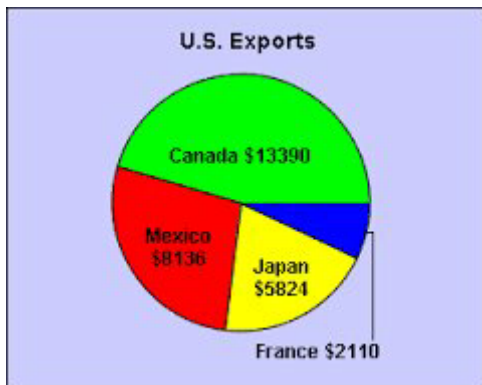
$$\text{Canada angle} = \frac{\text{amount of export}}{\text{total population}} \times 360$$

$$\frac{13390}{29460} \times 360 = 163.62^\circ$$

$$\text{Mexico} = \frac{8136}{29460} \times 360 = 99.42^\circ$$

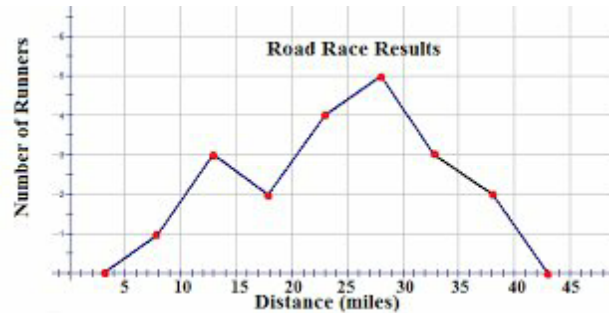
$$\text{Japan} = \frac{5824}{29460} \times 360 = 71.16^\circ$$

$$\text{France} = \frac{2110}{29460} \times 360 = 25.78^\circ$$



## Line graph

Data represented using lines



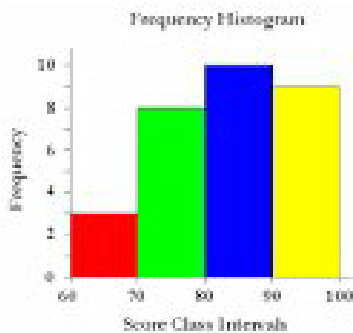
## Histograms

Frequency in each class is represented by a rectangular bar whose area is proportional to the frequency. When the bars are of the same width the height of the rectangle is proportional to the frequency.

### Note;

The bars are joined together.

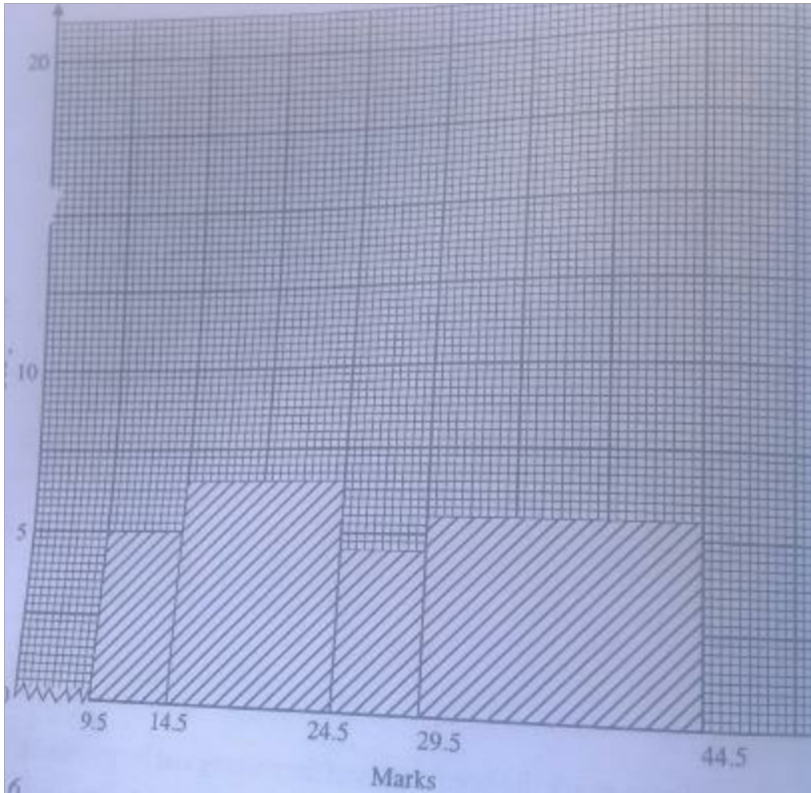
The class boundaries mark the boundaries of the rectangular bars in the histogram



Histograms can also be drawn when the class interval is not the same

The below information can be represented in a histogram as below

<b>Marks</b>	10- 14	15- 24	25 - 29	30 - 44
<b>No.of students</b>	5	16	4	15



**Note ;**

When the class is doubled the frequency is halved

**Frequency polygon**

It is obtained by plotting the frequency

against mid points.



End of topic

Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

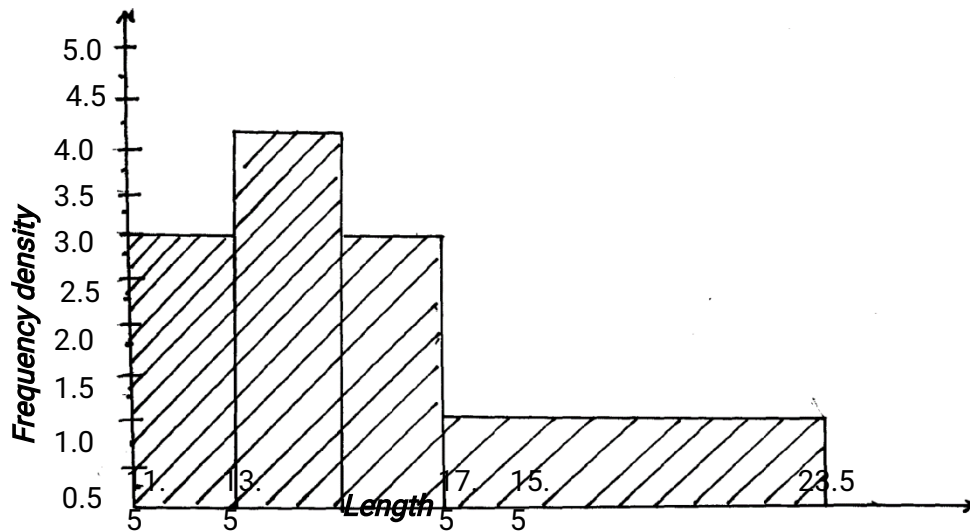
## Past KCSE Questions on the topic.

1. The height of 36 students in a class was recorded to the nearest centimeters as follows.

148 159 163 158 166 155 155 179 158 155 171 172 156  
 161 160 165 157 165 175 173 172 178 159 168 160 167 147  
 168 172 157 165 154 170 157 162 173

(a) Make a grouped table with 145.5 as lower class limit and class width of 5. (4mks)

2. Below is a histogram, draw.



Use the histogram above to complete the frequency table below:

Length	Frequency
$11.5 \leq x \leq 13.5$	

$13.5 \leq x \leq 15.5$	
$15.5 \leq x \leq 17.5$	
$17.5 \leq x \leq 23.5$	

3. Kambui salary as follows:

spent her

Food	40%
Transport	10%
Education	20%
Clothing	20%
Rent	10%

Draw a pie chart to represent the above information

4. The examination marks in a mathematics test for 60 students were as follows;-

60	54	34	83	52	74	61	27	65	22
70	71	47	60	63	59	58	46	39	35
69	42	53	74	92	27	39	41	49	54
25	51	71	59	68	73	90	88	93	85
46	82	58	85	61	69	24	40	88	34
30	26	17	15	80	90	65	55	69	89
<b>Class</b>	<b>Tally</b>	<b>Frequency</b>			<b>Upper class limit</b>				
10-29									
30-39									
40-69									
70-74									
75-89									
90-99									

From the table;

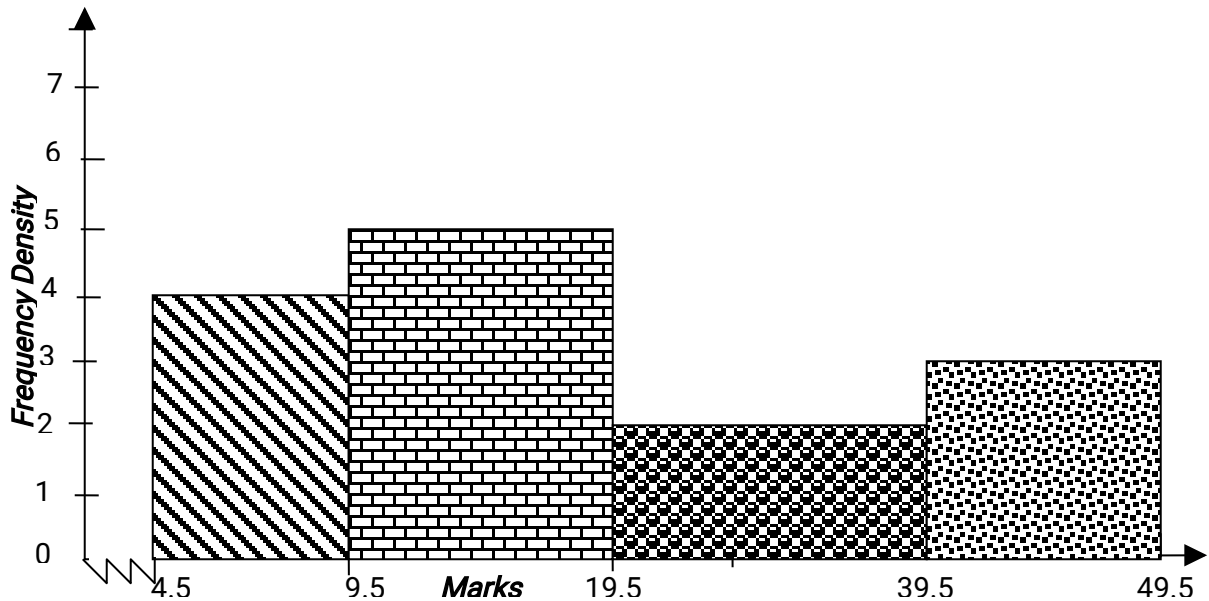
- (a) State the modal class
- (b) On the grid provided , draw a histogram to represent the above information

5. The marks scored by 200 from 4 students of a school were recorded as in the table below.

<b>Marks</b>	41 – 50	51 – 55	56 – 65	66 – 70	71 – 85
<b>Frequency</b>	21	62	55	50	12

- a.) On the graph paper provided, draw a histogram to represent this information.
- b.) On the same diagram, construct a frequency polygon.
- c.) Use your histogram to estimate the modal mark.

6. The diagram below shows a histogram representing the marks obtained in a certain test:-



- (a) If the frequency of the first class is 20, prepare a frequency distribution table for the data
- (b) State the modal class
- (c) Estimate:
  - (i) The mean mark
  - (ii) The median mark

## CHAPTER FOURTY TWO

### ANGLE PROPERTIES OF A CIRCLE

#### Specific Objectives

By the end of the topic the learner should be able to:

- a.) Identify an arc, chord and segment
- b.) Relate and compute angle subtended by an arc at the circumference;
- c.) Relate and compute angle subtended by an arc at the centre and at the circumference
- d.) State the angle in the semi-circle
- e.) State the angle properties of a cyclic quadrilateral
- f.) Find and compute angles of a cyclic quadrilateral.

#### Content

- a.) Arc, chord and segment.

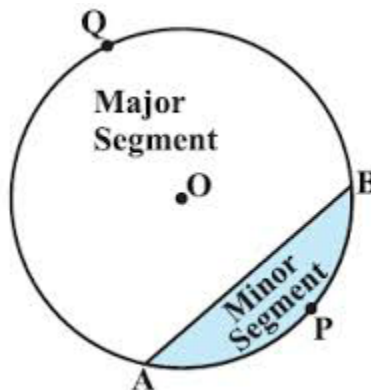
- b.) Angle subtended by the same arc at the circumference
- c.) Relationship between angle subtended at the centre and angle subtended on the circumference by the same arc
- d.) Angle in a semi-circle
- e.) Angle properties of a cyclic quadrilateral
- f.) Finding angles of a cyclic quadrilateral.

## Introduction

### Arc, Chord and Segment of a circle

#### Arc

Any part on the circumference of a circle is called an arc. We have the major arc and the minor Arc as shown below.

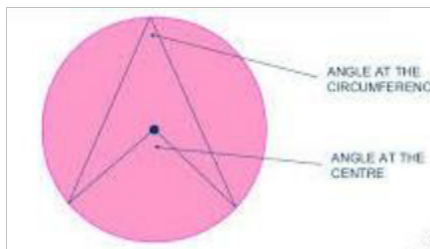
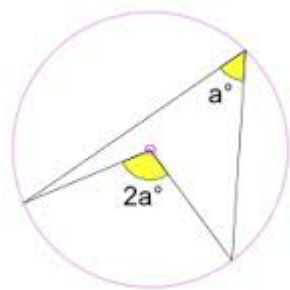
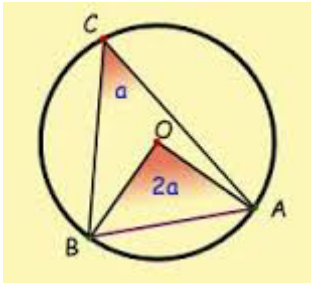


## Chord

A line joining any two points on the circumference. Chord divides a circle into two regions called segments, the larger one is called the major segment the smaller part is called the minor segment.

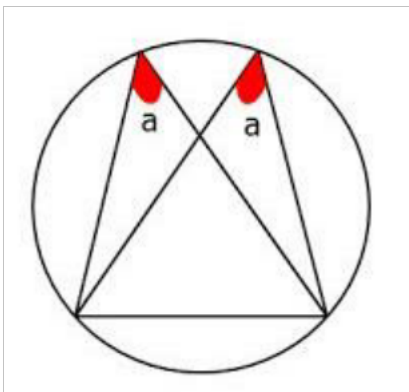
## Angle at the centre and Angle on the circumference

The angle which the chord subtends to the centre is twice that it subtends at any point on the circumference of the circle.



## Angle in the same segments

Angles subtended on the circumference by the same arc in the same segment are equal. Also note that equal arcs subtend equal angles on the circumference

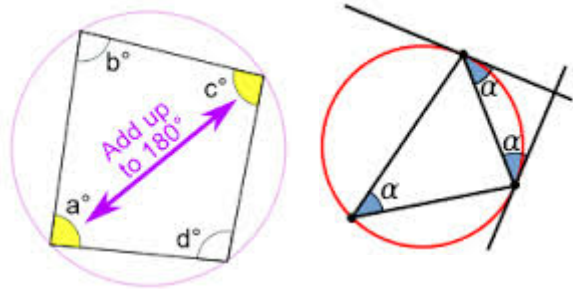


## Cyclic quadrilaterals

Quadrilateral with all the vertices lying on the circumference are called cyclic quadrilateral

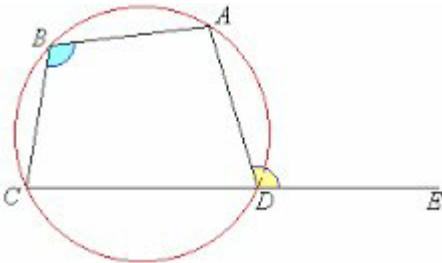
## Angle properties of cyclic quadrilateral

- ✓ The opposite angles of cyclic quadrilateral are supplementary hence they add up to  $180^{\circ}$ .
- ✓ If a side of quadrilateral is produced the interior angle is equal to the opposite exterior angle.



## Example

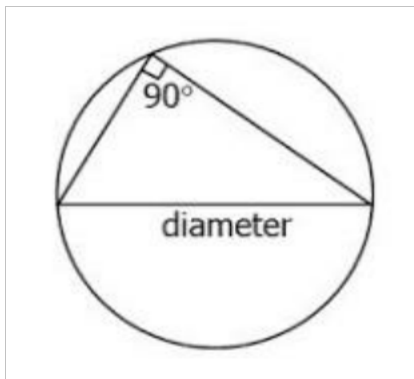
In the figure below  $\angle ADE = 120^{\circ}$  find  $\angle ABC$



## Solution

Using this rule, If a side of quadrilateral is produced the interior angle is equal to the opposite exterior angle.  
Find  $\angle ABC = 120^{\circ}$

Angles formed by the diameter to the circumference is always  $90^{\circ}$

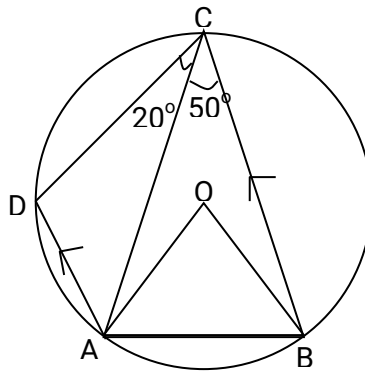


## Summary

- ✓ Angle in semicircle = right angle
- ✓ Angle at centre is twice than at circumference
- ✓ Angles in same segment are equal
- ✓ Angles in opposite segments are supplementary

## Example

1.) In the diagram, O is the centre of the circle and AD is parallel to BC. If angle  $ACB = 50^\circ$  and angle  $ACD = 20^\circ$ .



Calculate; (i)  $\angle OAB$

(ii)  $\angle ADC$

Solution i)  $\angle AOB = 2 \angle ACB$

$$= 100^\circ$$

$\angle OAB = \frac{180 - 100}{2}$  Base angles of Isosceles  $\Delta$

2

$$= 40^\circ$$

(ii)  $\angle BAD = 180^\circ - 70^\circ$

$$= 110$$

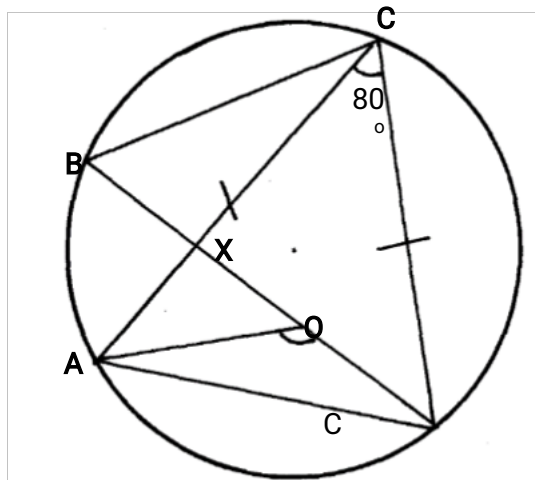
End of topic

Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

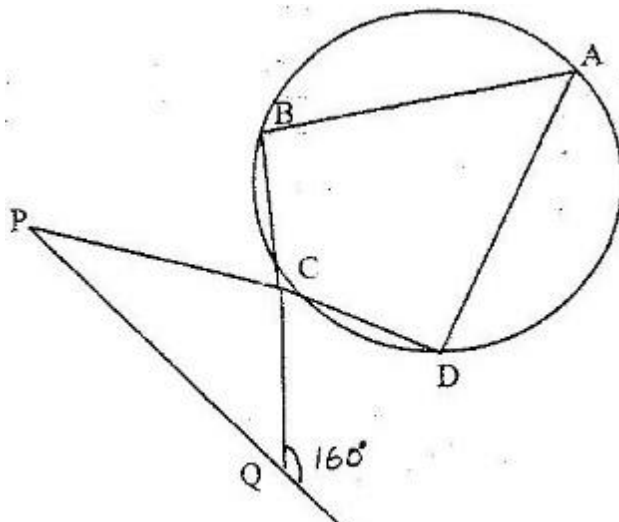
## Past KCSE Questions on the topic.

1. The figure below shows a circle centre  $O$  and a cyclic quadrilateral  $ABCD$ .  $AC = CD$ , angle  $ACD$  is  $80^\circ$  and  $BOD$  is a straight line. Giving reasons for your answer, find the size of :-

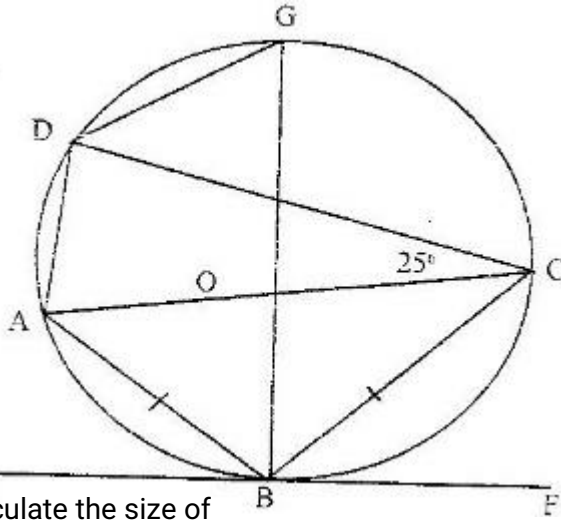


- (i) Angle  $ACB$
- (ii) Angle  $AOD$
- (iii) Angle  $CAB$
- (iv) Angle  $ABC$
- (v) Angle  $AXB$

- 1 In the figure below  $CP = CQ$  and  $\angle CQP = 160^\circ$ . If  $ABCD$  is a cyclic quadrilateral, find  $\angle BAD$ .

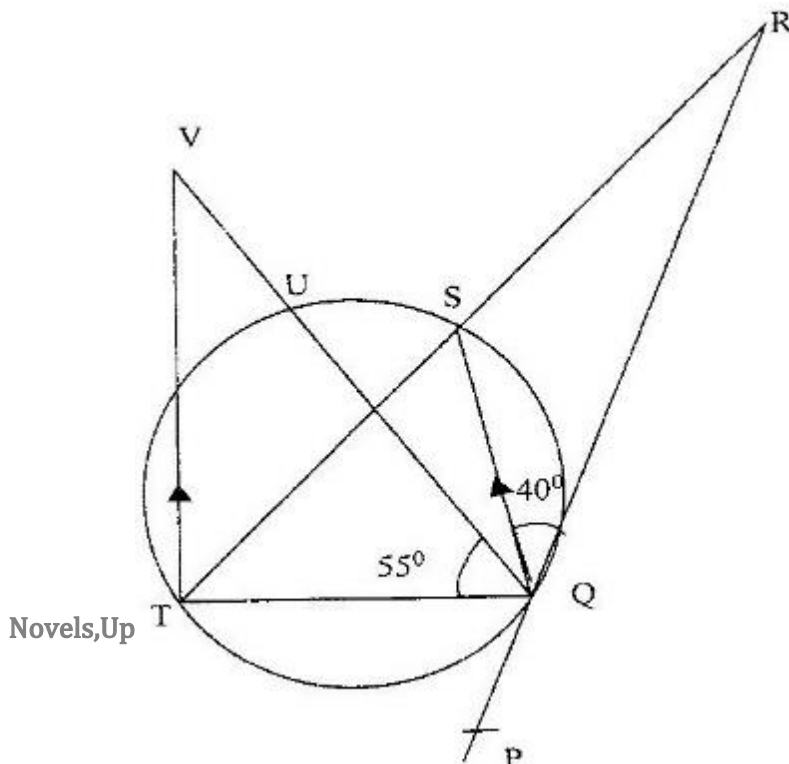


- 2 In the figure below  $AOC$  is a diameter of the circle centre  $O$ ;  $AB = BC$  and  $\angle ACD = 25^\circ$ ,  $EBF$  is a tangent to the circle at  $B$ .  $G$  is a point on the minor arc  $CD$ .



- (a) Calculate the size of
- (i)  $\angle BAD$
  - (ii) The Obtuse  $\angle BOD$
  - (iii)  $\angle BGD$
- (b) Show the  $\angle ABE = \angle CBF$ . Give reasons

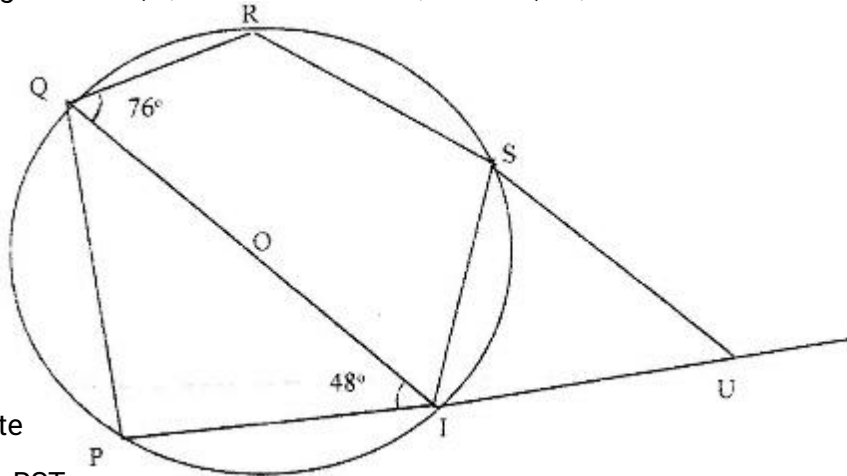
- 3 In the figure below  $PQR$  is the tangent to circle at  $Q$ .  $TS$  is a diameter and  $TSR$  and  $QUV$  are straight lines.  $QS$  is parallel to  $TV$ . Angles  $\angle SQR = 40^\circ$  and angle  $\angle TQV = 55^\circ$



Find the following angles, giving reasons for each answer

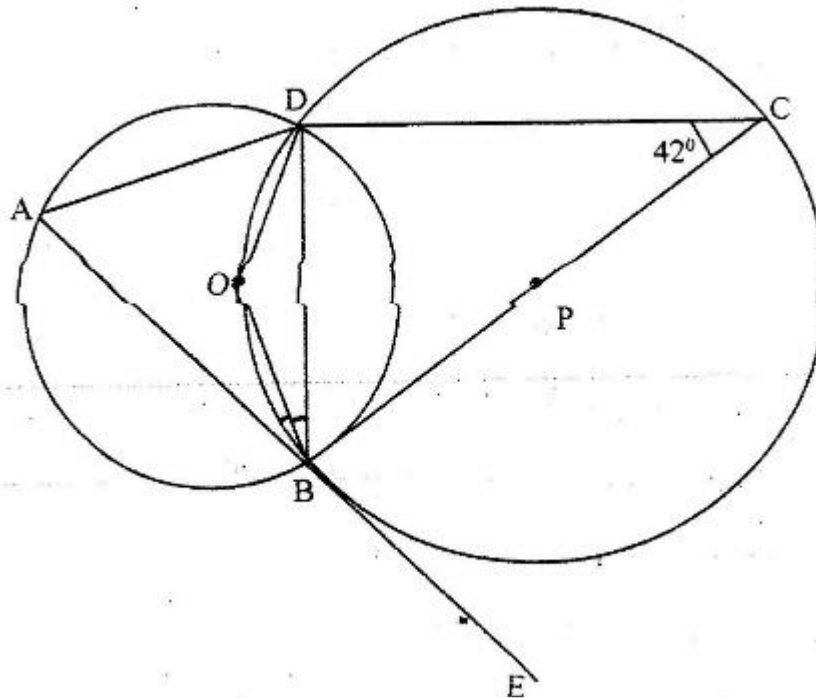
- (a)  $\angle QST$
- (b)  $\angle QRS$
- (c)  $\angle QVT$
- (d)  $\angle UTV$

4. In the figure below,  $QOT$  is a diameter.  $\angle QTR = 48^\circ$ ,  $\angle TQR = 76^\circ$  and  $\angle SRT = 37^\circ$



Calculate

- (a)  $\angle RST$
  - (b)  $\angle SUT$
  - (c) Obtuse  $\angle ROT$
5. In the figure below, points  $O$  and  $P$  are centers of intersecting circles  $ABD$  and  $BCD$  respectively. Line  $ABE$  is a tangent to circle  $BCD$  at  $B$ . Angle  $BCD = 42^\circ$



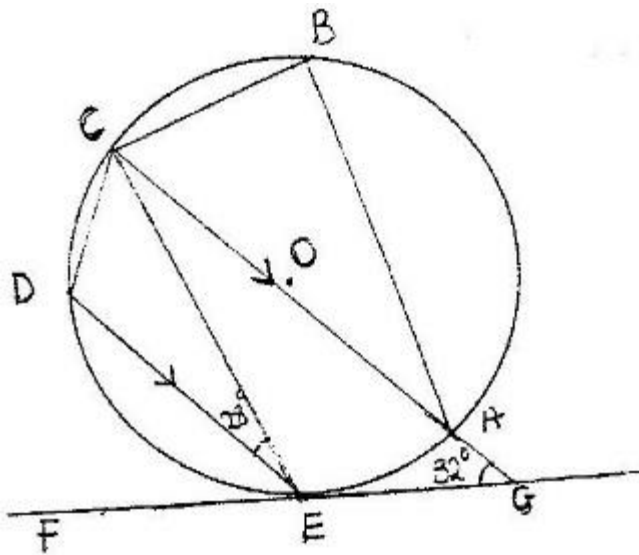
(a) Stating reasons, determine the size of

(i)  $\angle CBD$

(ii) Reflex  $\angle BOD$

(b) Show that  $\triangle ABD$  is isosceles

6. The diagram below shows a circle  $ABCDE$ . The line  $FEG$  is a tangent to the circle at point  $E$ . Line  $DE$  is parallel to  $CG$ ,  $\angle DEC = 28^\circ$  and  $\angle AGE = 32^\circ$

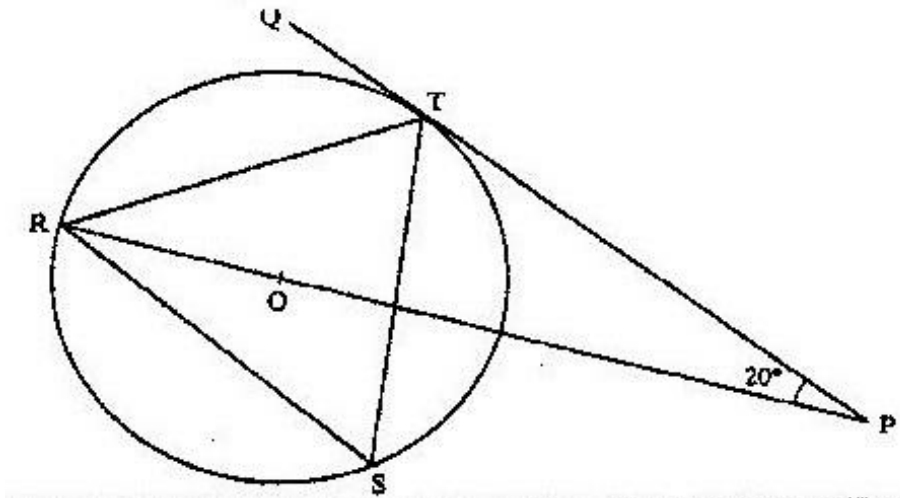


Calculate:

(a)  $\angle AEG$

(b)  $\angle ABC$

7. In the figure below  $R, T$  and  $S$  are points on a circle centre  $O$ .  $OPQ$  is a tangent to the circle at  $T$ .  $POR$  is a straight line and  $\angle QPR = 20^\circ$



Find the size of  $\angle RST$

## CHAPTER FOURTY THREE

# VECTORS

### Specific Objectives

By the end of the topic the learner should be able to:

- a.) Define vector and scalar
- b.) Use vector notation
- c.) Represent vectors both single and combined geometrically
- d.) Identify equivalent vectors
- e.) Add vectors
- f.) Multiply vectors by scalars
- g.) Define position vector and column vector
- h.) Find magnitude of a vector
- i.) Find mid-point of a vector
- j.) Define translation as a transformation.

### Content

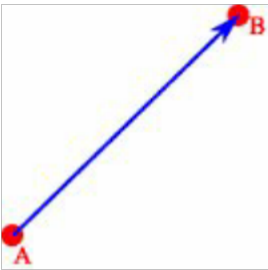
- a.) Vector and scalar quantities
- b.) Vector notation
- c.) Representation of vectors
- d.) Equivalent vectors
- e.) Addition of vectors
- f.) Multiplication of a vector by a scalar
- g.) Column vectors
- h.) Position vectors
- i.) Magnitude of a vector
- j.) Midpoint of a vector
- k.) Translation vector.

## Introduction

A **vector** is a quantity with both magnitude and direction, e.g. acceleration velocity and force. A quantity with magnitude only is called scalar quantity e.g. mass temperature and time.

## Representation of vectors

A vector can be presented by a directed line as shown below:



The direction of the vector is shown by the arrow.

Magnitude is the length of AB

Vector AB can be written as  $\vec{AB}$  or  $\overrightarrow{AB}$

Magnitude is denoted by  $|AB|$

A is the initial point and B the terminal point

## Equivalent vectors

Two or more vectors are said to be equivalent if they have:

- ✓ Equal magnitude
- ✓ The same direction.

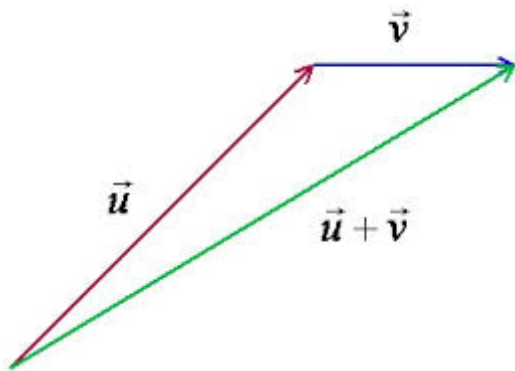
**Vector Representation**

- **Equal Vectors** - Two vectors are equal if they have the same magnitude AND the same direction as in (a). The line of action may be different as shown.
- **Negative Vectors** - Two vectors are negatives of each other if they have the same magnitude and opposite directions as in (b).

### Addition of vectors

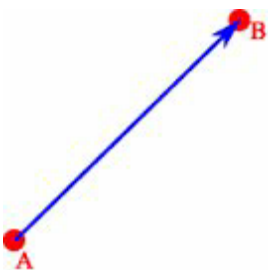
A movement on a straight line from point A to B can be represented using a vector. This movement is called displacement

Consider the displacement from  $\vec{u}$  followed by  $\vec{v}$



The resulting displacement is written as  $\vec{u} + \vec{v}$

### Zero vector



Consider a displacement from A to B and back to A. The total displacement is zero denoted by 0

This vector is called a Zero or null vector.

$$AB + BA = 0$$

If  $a + b = 0$  ,  $b = -a$  or  $a = -b$

## Multiplication of a vector by a scalar

### Positive Scalar

If  $AB = BC = CD = a$

A\_\_\_\_\_B\_\_\_\_\_C\_\_\_\_\_D>

$AD = a + a + a = 3a$

### Negative scalar

Subtraction of one vector from another is performed by adding the corresponding negative Vector. That is, if we seek  $a - b$  we form  $a + (-b)$ .

$$\begin{aligned} DA &= (-a) + (-a) + (-a) \\ &= -3a \end{aligned}$$

### The zero Scalar

When vector  $a$  is multiplied by 0, its magnitude is zero times that of  $a$ . The result is zero vector.

$$a \cdot 0 = 0 \cdot a = 0$$

### Multiplying a Vector by a Scalar

If  $k$  is any positive scalar and  $a$  is a vector then  $ka$  is a vector in the same direction as  $a$  but  $k$  times longer. If  $k$  is negative,  $ka$  is a vector in the opposite direction to  $a$  and  $k$  times longer.

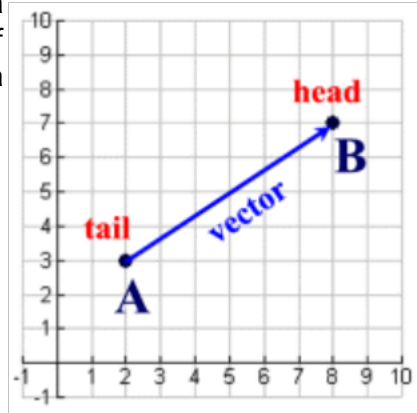
More illustrations.....

A vector is represented by a **directed line segment**, which is a segment with an arrow at one end indicating the direction of movement. Unlike a ray, a directed line segment has a specific length.

The direction is indicated by an arrow pointing from the **tail** (the initial point) to the **head** (the terminal point). If the tail is at point **A** and the head is at point **B**, the vector from **A** to **B** is written as:

notation:  $\overrightarrow{AB}$

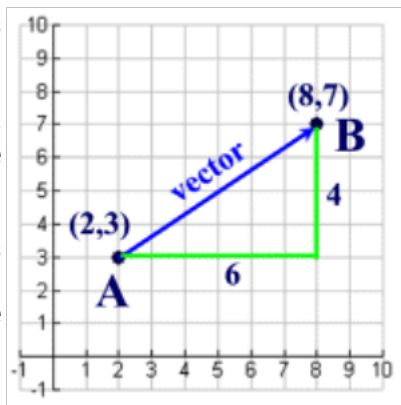
(Vectors may also be labeled as a single bold face letter, such as vector **v**.)



The **length** (magnitude) of a vector **v** is written **|v|**. Length is always a non-negative real number.

As you can see in the diagram at the right, the length of a vector can be found by forming a right triangle and utilizing the Pythagorean Theorem or by using the Distance Formula.

The vector at the right translates 6 units to the right and 4 units upward. The magnitude of the vector is  $2\sqrt{13}$  from the Pythagorean Theorem, or from the Distance Formula:



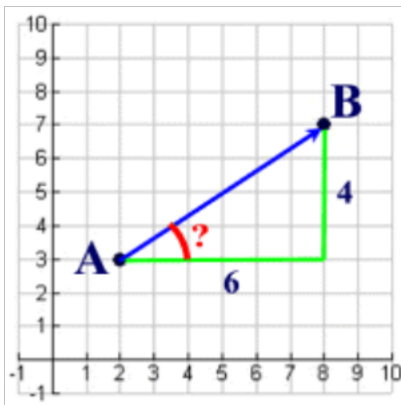
$$|\overrightarrow{AB}| = \sqrt{(8-2)^2 + (7-3)^2} = 2\sqrt{13}$$

The **direction** of a vector is determined by the angle it makes with a horizontal line.

In the diagram at the right, to find the direction of the vector (in degrees) we will utilize trigonometry. The tangent of the angle formed by the vector and the horizontal line (the one drawn parallel to the x-axis) is 4/6 (opposite/adjacent).

$$\tan \angle A = \frac{4}{6}$$

$$\tan^{-1}\left(\frac{4}{6}\right) \approx 33.7^\circ$$



A free vector is an infinite set of parallel directed line segments and can be thought of as a translation. Notice that the vectors in this translation which connect the pre-image vertices to

the image vertices are all parallel and are all the same length.

You may also hear the terms "displacement" vector or "translation" vector when working with translations.

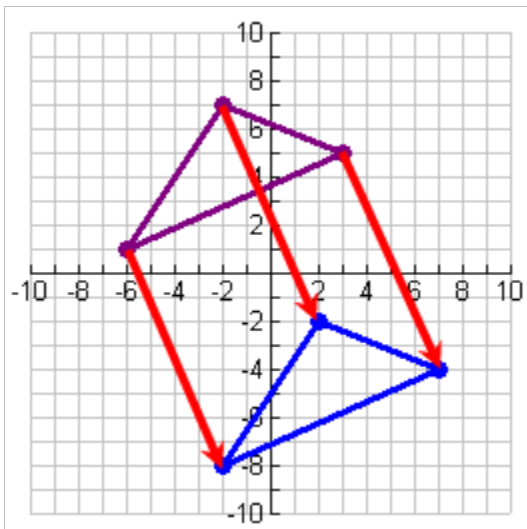
**Position vector:**

To each free vector (or translation), there corresponds a **position vector** which is the image of the **origin** under that translation.

Unlike a free vector, a position vector is "tied" or "fixed" to the origin. A position vector describes the spatial position of a point relative to the origin.

**TRANSLATION VECTOR**

Translation vector moves every point of an object by the same amount in the given vector direction. It can be simply be defined as the addition of a constant vector to every point.



**Translations and vectors:** The translation at the left shows a vector translating the top triangle 4 units to the right and 9 units downward. The notation for such vector movement may be written as:

$$\langle 4, -9 \rangle \text{ or } \begin{pmatrix} 4 \\ -9 \end{pmatrix}$$

Vectors such as those used in translations are what is known as **free vectors**. Any two vectors of the same length and parallel to each other are considered identical. They need not have the same initial and terminal points.

**Example**

The points A (-4 , 4 ) , B (-2 , 3 ) , C (-4 , 1 ) and D ( - 5 , 3 ) are vertices of a quadrilateral. If the quadrilateral is given the translation T defined by the vector  $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$  draw the quadrilateral ABCD and its image under T

**Solution**

$$OA^1 = \begin{pmatrix} -4 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \text{ so } A^1 \text{ is } ( 1 , 1)$$

$$OB^1 = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \text{ so } B^1 \text{ is } ( 3 , 0)$$

$$OC^1 = \begin{pmatrix} -4 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \text{ so } C^1 \text{ is } (1, -2)$$

$$OD^1 = \begin{pmatrix} -5 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \text{ so } D^1 \text{ is } (0, 0)$$

### Summary on vectors

#### Components of a Vector in 2 dimensions:

To get from A to B you would move:

2 units in the x direction (x-component)

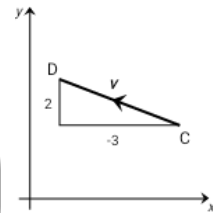
4 units in the y direction (y-component)

The components of the vector are these moves in the form of a column vector.

thus  $\overline{AB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$  or  $u = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$



Similarly:  $\overline{CD} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$  or  $v = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$



#### Magnitude of a Vector in 2 dimensions:

We write the magnitude of  $u$  as  $|u|$

$$u = \begin{pmatrix} x \\ y \end{pmatrix} \text{ then } |u| = \sqrt{x^2 + y^2}$$

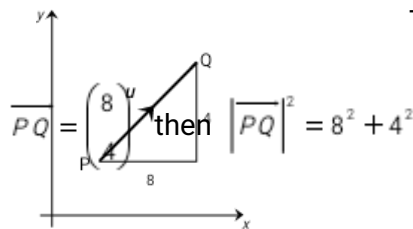
The magnitude of a vector is the length of the directed line segment which represents it.

Use Pythagoras' Theorem

to calculate the length of the vector.

The magnitude of vector  $u$  is  $|u|$  (the length of PQ)

The length of PQ is written as  $|\overline{PQ}|$



and so  $|\overline{PQ}| = \sqrt{8^2 + 4^2} = \sqrt{80} = 8.9$

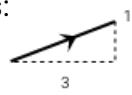
Examples:

1. Draw a directed line segment representing  $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

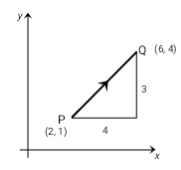
2.  $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$  and P is (2, 1), find coordinates of Q

3. P is (1, 3) and Q is (4, 1) find  $\overrightarrow{PQ}$

Solutions:

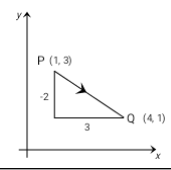


1.



2. Q is (2 + 4, 1 + 3) → Q(6, 4)

3.  $\overrightarrow{PQ} = \begin{pmatrix} 4-1 \\ 1-3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$



**Vector:**

A quantity which has magnitude and direction.

**Scalar:**

A quantity which has magnitude only.

**Examples:**

Displacement, force, velocity, acceleration.

**Examples:**

Temperature, work, width, height, length, time of day.

End of topic

Did you understand everything?

If not ask a teacher, friends or anybody and make sure you understand before going to sleep!

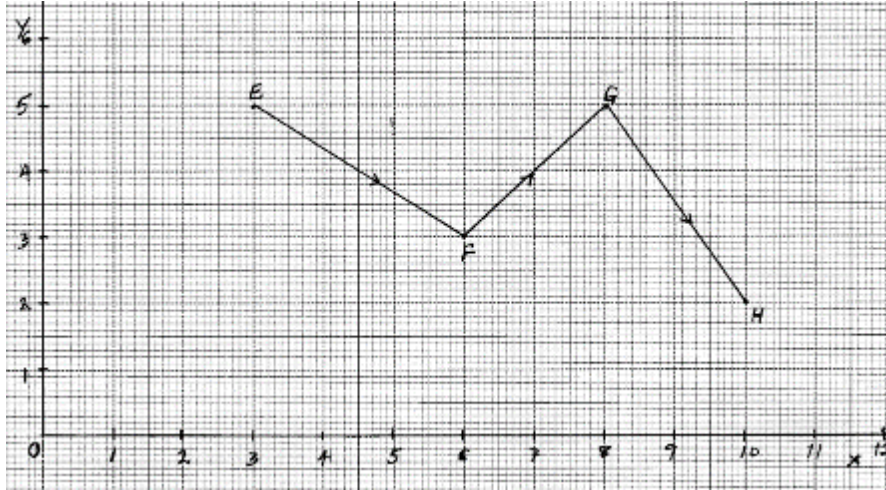
## Past KCSE Questions on the topic.

1. Given that  $4\vec{p} - 3\vec{q} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$  and  $\vec{p} + 2\vec{q} = \begin{pmatrix} -14 \\ 15 \end{pmatrix}$  find
- a) (i)  $\vec{p}$  and  $\vec{q}$  (3 mks)
- (ii)  $|\vec{p} + 2\vec{q}|$  (3 mks)
- (b) Show that A (1, -1), B (3, 5) and C (5, 11) are collinear (4 mks)
2. Given the column vectors  $\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ ,  $\vec{b} = \begin{pmatrix} 6 \\ -3 \\ 9 \end{pmatrix}$ ,  $\vec{c} = \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix}$  and that  $\vec{p} = 2\vec{a} - \frac{1}{3}\vec{b} + \vec{c}$
- (c) (i) Express  $\vec{p}$  as a column vector (2mks)
- (d) (ii) Determine the magnitude of  $\vec{p}$  (1mk)
3. Given the points P(-6, -3), Q(-2, -1) and R(6, 3) express PQ and QR as column vectors. Hence show that the points P, Q and R are collinear. (3mks)
4. The position vectors of points x and y are  $x = 2i + j - 3k$  and  $y = 3i + 2j - 2k$  respectively. Find  $\vec{xy}$  as a column vector (2 mks)
5. Given that  $\vec{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ,  $\vec{b} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$ ,  $\vec{c} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$  and  $\vec{p} = 2\vec{a} + \vec{b} - 3\vec{c}$ . find  $|\vec{p}|$  (3mks)
6. The position vectors of A and B are  $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$  and  $\begin{pmatrix} 8 \\ -7 \end{pmatrix}$  respectively. Find the coordinates of M

which divides AB in the ratio 1:2.

(3 marks)

7. The diagram shows the graph of vectors  $\vec{EF}$ ,  $\vec{FG}$  and  $\vec{GH}$ .



Find the column vectors;

(a)  $\vec{EH}$  (1mk)

(b)  $|\vec{EH}|$  (2mks)

8.  $\vec{OA} = 2\vec{i} - 4\vec{k}$  and  $\vec{OB} = -2\vec{i} + \vec{j} - \vec{k}$ . Find  $|\vec{AB}|$  (2mks)

9. Find scalars  $m$  and  $n$  such that

$$m \begin{pmatrix} 4 \\ 3 \end{pmatrix} + n \begin{pmatrix} -3 \\ 2 \\ 8 \end{pmatrix} = 5 \begin{pmatrix} \phantom{0} \\ \phantom{0} \\ \phantom{0} \end{pmatrix}$$

10. Given that  $\vec{p} = 2\vec{i} - \vec{j} + \vec{k}$  and  $\vec{q} = \vec{i} + \vec{j} + 2\vec{k}$ , determine

(a.)  $|\vec{p} + \vec{q}|$  (1 mk)

(b)  $|\vec{p} - 2\vec{q}|$  (2 mks)